Stress Tests: Frequency vs. Strength*

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Abstract

This paper studies how the stress test design (in particular its strength and frequency) affects its effectiveness in providing information to persuade the bank's stakeholders to coordinate on not "attacking" the bank to decrease the probability of bank failure during distress. The stakeholders are privately informed and move sequentially. We characterize all robustly persuasive stress tests, under which, even in the worst equilibrium, all bank stakeholders disregard their private information and perfectly coordinate their actions based on the test results ("pass" or "fail"). We show that testing the bank more frequently can substitute the role of an increased test strength in making the stress test result persuasive. We then characterize the optimal frequency and investigate how it depends on macroeconomic conditions, the bank's idiosyncratic characteristics, as well as the endogenous maturity choices of banks. We further examine how other regulatory measures may complement the stress test policy.

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1 Introduction

Runs by institutional investors such as money market funds contributed to the onset of the global financial crisis in 2008 (Brunnermeier, 2009). Runs are rooted in the coordination problem among the creditors (Diamond and Dybvig, 1983), and, therefore, creditors' information and uncertainty about the bank's fundamental, as well as other creditors' decisions, are critical in determining the run and resulting likelihood of a crisis (Goldstein and Pauzner, 2005; Rochet and Vives, 2004). Following the crisis, bank supervisors implemented new regulatory measures —- stress tests, which provide critical information to the market in order to avert runs and maintain financial stability. Some examples of such a policy include the Dodd Frank Act Stress Tests (DFAST) conducted by the Federal Reserve and EU-wide stress tests conducted by the European Banking Authority (EBA) where the supervisors evaluate the banks' capital under adverse scenarios and publicly disclose the results.

In practice, bank stress tests have a pass/fail design and are conducted typically on a monthly, quarterly, or semi-annual basis. While the other dimensions of stress test design (e.g., binary result) have been well investigated theoretically (See, for instance, Goldstein and Huang (2016) and Inostroza and Pavan (2021)), little attention has been drawn to consider the optimal frequency of stress test. In this paper, we set out to fill this gap by studying the optimal design of stress test frequency in conjunction with its strength.

We build a model of dynamic coordination to examine the impact of frequent stress tests, which publicly disclose information at regular time intervals. More specifically, in our model, the bank may face a shock in the future. The distress lasts for a fixed period of time, normalized to $[0, 1]$, and during this time, a mass 1 of agents sequentially make their decisions. The shock may trigger a run, which further worsens the bank's fundamentals and may lead to bank failure. The agents have incomplete information regarding the bank's fundamentals, and they do not observe the past actions, while the stress test reveals partial information regarding the fundamental and past actions.

Let us start with the benchmark — a one-time stress test policy, which at the beginning of the distress (time 0) discloses whether the bank's fundamental θ is high enough to sustain k proportion of attack from its stakeholders. Notice that Although the agents move sequentially, since they do not receive any new information over time, the game is the same if they were moving simultaneously. It is well known that in such a game, the worst equilibrium is such that an agent attacks if either the bank fails the stress test or the bank passes the stress test but the private signal $s_i < \hat{s}$, for some cutoff $\hat{s}$. This means there exists a cutoff $\hat{\theta} \geq k$ such that the bank will survive in the end iff $\theta \geq \hat{\theta}$.

Notice if all the agents ignore their private signals and perfectly coordinate their actions based on the stress test result, then an agent will do the same. Therefore, there is always an equilibrium with $\hat{\theta} = k$. If $\hat{\theta} > k$, then it must be that the corresponding marginal agent (with signal $\hat{s}$) is indifferent between attacking and not attacking. We are interested in the worst equilibrium, that is the equilibrium with highest $\hat{\theta}$.

To see how more tests may help, consider two tests — one at the beginning and one after half of the agents have made their decisions. Each test discloses whether the residual fundamental strength can sustain k proportion of attack from the agents yet to make a decision. We will call the agents moving after the second test group 2 and the agents moving before the second test group 1. It is intuitive that he worst equilibrium will be as follows. An agent in group $n = 1, 2$ attacks if the bank fails any of the tests so far, or if it passes all the tests so far but $s_i < \hat{s}_n$. Given the cutoff strategies there exists $k \leq \underline{\theta}_1 \leq \hat{\theta}$ such that the bank survives iff $\theta \geq \hat{\theta}$. However, unlike under single test, the bank passes the first test but fails the second test when $\theta \in [k, \underline{\theta}_1)$.

Since $\underline{\theta}_1 \geq k$, the second group learns more than the first group when they see that the bank has passed both tests. This makes them more optimistic about the bank's survival, and hence, if the bank passes both tests, there is less attack from group 2 compared to group 1. For any cutoff $\hat{\theta}$, the corresponding marginal agent $\hat{s}$ under single test policy is such that the aggregate attack at $\theta = \hat{\theta}$ is exactly $\hat{\theta}$. In contrast, under two tests, the aggregate attack from group 1 is higher and group 2 is lower than $\hat{\theta}$. This requires

$$\hat{s}_1 \geq \hat{s} \geq \hat{s}_2.$$

However, this means that the marginal agent in group 1 is more optimistic about the survival of the bank than the marginal agent under single test policy.

Finally, we show that if θ^* is the highest equilibrium fundamental cutoff under a single test policy, then for any $\hat{\theta} > \theta^*$, the corresponding marginal agent $\hat{s}$ strictly prefers not attacking, which implies the same holds for the marginal agent ($\hat{s}_1$) in group 1 under two-tests policy. Therefore, the worst θ^* can only be lower under two tests. In fact, we show that it is strictly lower and the argument follows from the fact that the second group learns strictly more.

A general stress test policy $\Gamma = (N, k)$ with frequency N and strength k, conducts N tests at regular interval of $1/N$ starting from 0, and publicly discloses whether the per-capita residual fundamental is above the threshold k (pass) or not (fail). Given the stress test policy, we look into perfect Bayesian equilibrium in monotone strategies and focus on the worst equilibrium — the one with the maximum probability of failure. Consider an agent who moves after the n-th test but before the next test (if there is one). We refer to these agents as group n agents. In the worst equilibrium, an agent in group n runs if (1) the bank fails any of the tests so far, (2) the bank passes all the tests so far but the private signal $s_i < s_n^*$. Accordingly, there exists a weakly increasing sequence

$$k = \underline{\theta}_0 \leq \underline{\theta}_1 \leq \ldots \leq \underline{\theta}_{N-1} \leq \underline{\theta}_N = \theta^*$$

such that the bank passes the first n tests iff $\theta \geq \underline{\theta}_{n-1}$, and survives in the end iff $\theta \geq \theta^*$.

In equilibrium, either group n is persuaded, that is, all the agents in group n prefer not running;

or they are not persuaded, in which case, the marginal agent s_n^* in group n is indifferent between running and not running. Since, the agents moving later are more optimistic if the bank passes all the tests thus far, they are less aggressive. This implies there is a last group that is not persuaded. Notice that for any stress test policy $\gamma = (N, k)$, if all the groups are persuaded, then an agent will always follow the stress test result rather than her private signal. Therefore, there is always an equilibrium with $\hat{\theta} = k$. We call it the obedient equilibrium. However, there can be many disobedient equilibrium. A stress test is *robustly persuasive* if in the worst equilibrium, all the groups are persuaded.

Our main result is that for any frequency of stress test N, there is a threshold strength $k^*(N)$ such that a stress test $\Gamma = (N, k)$ is robustly persuasive iff $k > k^*(N)$. Moreover, $k^*(N)$ is decreasing and discrete convex.

Notice that a future stress test discloses information based on the endogenous attack from the earlier groups, while the agents in the earlier groups decide whether to attack based on their expectation of the effectiveness the future stress tests. Therefore, for any stress test policy $\Gamma = (N, k)$, the N thresholds $(s_n^*)_{n=1}^N$ in the worst equilibrium are jointly determined. For any stress test policy $\Gamma = (N, k)$, we find a lower bound on the belief of the marginal agent in the last non-persuaded group that the regime will survive — $G(x, N, k)$, where x is the proportion of attack from this group. We show that when $k > k^*(N)$ this marginal agent strictly prefers not attacking for any x, and hence, there cannot be a disobedient equilibrium. Moreover, if $k \leq k^*(N)$, we can always construct an equilibrium where at least the first group is not persuaded. For higher N, assuming the the later groups will be persuaded, the excess fundamental required to pass the next test is lower, making $G(x, N, k)$ increasing in N, which in turn makes $k^*(N)$ decreasing in N. We further show that $G(x, N, k)$ is quasi-concave in (N, k) making $k^*(N)$ discrete convex.

Our result generalizes the findings in Goldstein and Huang (2016) and Basak and Zhou (2020). Goldstein and Huang (2016) builds a static model and shows that the optimal stress test is a binary public signal (pass or fail). This can be mapped to our model when restricting it to a one-time disclosure, i.e., $N = 1$. The author shows there exists $k^*(1)$ such that the test is robustly persuasive as long as $k \geq k^*(1)$. Basak and Zhou (2020) show that when the debt structure of a bank is sufficiently asynchronous, then the viability news is enough to avoid liquidity crisis caused by panic-based runs. The public information of the continued viability of the bank can be interpreted as a stress test with $k = 0$. Given sufficiently asynchronous debt structure, say $N \geq N_0$, the required strength of test is $k^*(N_0) = 0$, so that the continued viability news, by itself, is strong enough to be persuasive. Our result nest these findings and give the complete characterization of robustly persuasive stress tests for any frequency N.

The above results give us the trade-off between frequency and strength of stress tests. A stronger test is harder to pass, and the regulator does not need to make the tests as strong if she can run more tests. This means the bank is more likely to pass the test. This accounts for the marginal benefit of increasing the frequency of tests. However, running more tests could be costly. Assuming a convex cost, this trade-off helps us uniquely determine the optimal stress test policy.

This paper shows why frequency is an important dimension in designing the stress test policy. To understand the policy implication, suppose that the market conditions worsens making it difficult for the regulator to dissuade the agents from running. A regulator who cannot conduct more tests will increase the strength of the stress tests under worse market conditions. However, if she can increase the frequency, the optimal policy could be more frequent but more lenient tests. We provide a numerical example illustrating this point in Section 6. Recently, European Securities Market Authority (ESMA) issued guidelines for liquidity stress testing of Alternative Investment Funds (AIFs) where funds are encouraged to conduct more frequent tests when there is a higher risk of the adverse shock. The Federal Reserve in its stress testing and monitoring framework categorizes banks based on their systemic importance and tests the more systemically important bank holding companies more frequently. We show that these observations are consistent with our theory.

It is important to note that this theory of frequent tests is based on the assumption that the agents move sequentially. If all the agents move simultaneously, then the regulator cannot disclose any new information by conducting more than one test. This sequential move assumption is motivated by the fact that during a distress, banks' debts mature asynchronously, and creditor run occurs in a dynamic manner (He and Xiong, 2012). For simplicity, in our benchmark setup, we assume that when the regulator conducts multiple tests, there are always some debts maturing between two tests. However, in practice, the bank may have debts maturing at M different dates. Therefore, if the regulator conducts more than M tests, this assumption is no longer true. In this sense, the bank's debt structure M imposes an upper bound on how many effective tests the regulator can run. To understand the effect of this constraint, we extend our model by introducing a first stage where the bank optimally chooses the asynchronicity of the debt maturity structure M. Since the bank does not internalize the social cost its failure imposes on the financial system, in equilibrium, it chooses a debt structure that is less asynchronous than socially optimal, while the regulator chooses the optimal frequency same as M and conducts the tests right before each maturity dates. We also show that alternative policies such as Liquidity Coverage Ratio (LCR) may make the bank's choice more aligned with the socially optimal, and thus, improve social welfare.

The rest of the paper is organized as follows. We begin with setting up the model and describing the stress test policy in Section 2, in Section 3 with the help of an example we explain how more frequent tests help in dissuading the agents from running, Section 4 and Section 5 provide a formal exposition of our main results. In Section 6 we describe some regulatory policy implications of our theory. In Section 7 we present an application of our theory for financial institutions with asynchronous debt structure and the constraint it imposes on the frequency of stress tests, followed by the conclusion.

2 Model

The economy consists of a bank, a continuum of stakeholders (agents) of banks and a regulator. An adverse shock may arrive at a future date. The bank's stakeholders have incomplete information regarding the severity of the shock and the resulting bank's asset quality. Stakeholders may *attack* (such as stop lending to the bank, increase the collateral requirements for the bank's borrowing, refuse to invest in the bank or short the bank equities), which can further deteriorate the bank's balance sheet and possibly result in bank failure. The regulator conducts *stress tests* to provide valuable information publicly to the stakeholders in order to mitigate the endogenous attack and promote financial stability. We focus on two aspects of the tests: (1) *frequency* — how often to conduct the tests, (2) *strength* — how difficult it is to pass the tests. Below, we describe the details of our model.

2.1 The bank and its stakeholders

An adverse shock arrives at some future date T_0 with probability $\pi \in (0, 1)$. Upon its arrival, we denote the fundamental of bank by a random variable θ. The arrival of this adverse shock is publicly observable but the realization of θ is not. We assume that θ is exogenous and that nature draws θ from a commonly known prior $\mathcal{U}[\underline{\theta}, \overline{\theta}]$. The larger the shock the lower the θ. We normalize the duration of the distress to 1. If the bank can survive the attack between T_0 to $T_1 = T_0 + 1$, then the bank is "out of woods."

A unit mass of stakeholders referred to as agents, indexed by $i \in [0, 1]$, move sequentially: each agent i decides of whether to attack ($a_i = 1$) or not ($a_i = 0$) at time $T_0 + i$. Without loss of generality, we normalize T_0 to 0 and therefore, the distress time is from $t = 0$ to 1.[1]

Notice that if the agents move simultaneously, rather than sequentially, then a regulator cannot disclose any new information by conducting more than one test, making the concept of frequent test meaningless. The sequential move naturally arises when bank has asynchronous debt structure, and the creditors decide whether to rollover when their debt matures. For simplicity of exposition, in our benchmark setup, we do not explicitly model the bank's choice of debt structure. In Section 7, we consider a bank that optimally choose the debt structure, and show how the result can be extended, and what are the implications of this endogenous debt maturity choice.

[1]In practice, the shock arrival time T_0 can be stochastic. However, whether it is deterministic or stochastic, or the precise value of T_0 does not matter in our model as our analysis focuses on the consequences after the arrival of the shock.

2.2 Deterioration of bank's fundamental

If the shock arrives, nature selects the bank's fundamental θ. The bank's fundamental deteriorates iff agents attack the bank. We model the deterioration of the fundamental as follows. The initial fundamental is $\theta^0 = \theta$ and the residual fundamental after time $t \in [0,1]$ is

$$\theta^t \equiv \theta^0 - w^t = \theta - \int_{i=0}^{t} \mathbb{1}\{a_i = 1\}\mathrm{d}i, \tag{1}$$

in which $w^t \in [0,t]$ is the cumulative attack until time t. If $\theta^1 = \theta - w^1 \geq 0$, the bank survives the exogenous shock and all endogenous attack; otherwise, the bank fails.

Dominance Regions It is possible that the shock is so severe —i.e., $\theta < 0$ — that the bank will fail even if no agent attacks. It is also possible that the shock is so mild —i.e., $\theta \geq 1$ — that the bank survives even when all agents attack. We call $[\underline{\theta}, 0)$ the lower dominance region and $[1, \overline{\theta}]$ the upper dominance region.[2]

2.3 Stress Test: A disclosure policy

The regulator commits to a stress test policy $\Gamma = (N, k)$, which is characterized by its frequency $N \in \mathbb{Z}^+$ and its strength $k \in [0,1]$. The regulator tests the bank's fundamental N times at uniform intervals of length $t_N \equiv \frac{1}{N}$, and each test has a per-capita strength $k \in [0,1]$.

Definition 1 (Stress Test Policy) *A stress test policy $\Gamma = (N, k)$ sends a public binary signal ζ_n at time $(n-1)t_N$ for $n = 1, 2, \ldots N$. The n-th test reports at time $t = (n-1)t_N$ whether the bank's per-capita fundamental is greater than k ("pass", $\zeta_n = \mathsf{P}$) or not ("fail", $\zeta_n = \mathsf{F}$); that is,*

$$\zeta_n \equiv \begin{cases} \mathsf{P}, & \text{if } \frac{\theta^{(n-1)t_N}}{1-(n-1)t_N} \geq k \\ \mathsf{F}, & \text{otherwise} \end{cases} \tag{2}$$

At any time t, if a stress test of strength $k \in [0,1]$ is conducted, then the regulator tests whether or not the bank's fundamental at time t is sufficient to sustain the attack from k fraction of remaining agents who have not yet made their decisions; that is, whether or not $\theta^t \geq k(1-t)$. Under a frequency of N, these tests are scheduled at $t = (n-1)t_N$ for $n = 1, 2, \ldots, N$. Unlike the first test, which tests the exogenous fundamental θ at time 0, each subsequent test discloses some partial information about the exogenous fundamental θ as well as the endogenous attack that occurred before the test (excluding the exact time of disclosure).

[2]We treat $\underline{\theta}$ and $\overline{\theta}$ as primitives. In Section 7, we consider alternative regulatory policies that can affect this range of possible θ.

Under a policy $\Gamma = (N, k)$, the agents who move at time $[(n-1)t_N, nt_N)$ see the same stress tests results. We refer to these agents as *group* n agents, for $n = 1, 2, \ldots, N$. Put differently, agent $i \in [0, 1)$ belongs to group $\lfloor iN \rfloor + 1$, and agent $i = 1$ belongs to group N.

2.4 Agents' Problem

After the regulator commits to a policy Γ, the agents play a dynamic coordination game, in which they sequentially decide whether or not to attack. If an agent believes that the bank will fail (will not fail) for sure, then attacking (not attacking) would be an optimal choice. As agents are uncertain about whether the bank can survive the exogenous shock as well as the endogenous attack, we assume that each agent would not attack if and only if they believe that the bank will survive in the end with a probability at least p, for some $p \in (0, 1)$. The parameter p captures the willingness to attack. When p is higher, agents are more inclined to attack.[3,4]

Exogenous Private Information Upon the arrival of the liquidity shock, each agent receives a noisy private signal regarding the bank's fundamental,

$$s_i = \theta + \sigma\epsilon_i,$$

where the noises $(\epsilon_i)_i$ are independent across agents conditional on θ and identically distributed according to $\epsilon_i \sim F$. We assume that F has support $[-1/2, 1/2]$, and it admits a density f, which is symmetric around 0, and $f(x) > 0$ for $x \in [-0.5, 0.5]$ and 0 otherwise. σ scales the noisiness of the private information. As is standard in the global games literature, we assume $\underline{\theta} < -\sigma$ and $\overline{\theta} > 1+\sigma$. Under this assumption, the posterior belief of agent with any private signal $s_i \in \mathbb{S} = [-\frac{\sigma}{2}, 1 + \frac{\sigma}{2}]$ is simply $F(\frac{\theta - s_i}{\sigma})$.

Assumption 1 *$f(.)$ is log-concave.*

[3] To see where the model primitive p comes from, consider the following example (as in Rochet and Vives (2004)). The payoff difference from rolling over and withdrawing is

$$\Delta u(a_{-i}, \theta) = u(a_i = 1, a_{-i}, \theta) - u_1(a_i = 0, a_{-i}, \theta) = b\mathbb{1}(\theta^1 \geq 0) - c\mathbb{1}(\theta^1 < 0),$$

where $b = r - 1 > 0$ is the benefit she gets if the bank survives in the end (promised return r rather than 1) and $c = 1 - d > 0$ is the cost she bears if the bank fails in the end (recovery value d rather than 1). Under this payoff specification, an agent would roll over instead of withdrawing if and only if she believes that the probability of bank survival is at least $p \equiv \frac{c}{b+c} = \frac{1-d}{r-d} \in (0, 1)$. It is worth noting that, in practice, the payoff the agents receive when the bank fails can be more nuanced (which may depend on the exact amount of withdrawal as well as the bank's fundamental θ). As long as there is a threshold belief p that can convince the agents to roll over, the exact specification plays no role for our main analysis.

[4] Notice that the agents only decide whether to attack or not. We assume that agents cannot trade their claims on the bank's assets among themselves or in any market. In practice, one can think of this assumption as if the market for these claims has broken down in the aftermath of the adverse shock, e.g. the frozen inter-bank markets in 2008.

This assumption gives us that the belief of an agent with the higher signal is ranked higher according to the likelihood ratio. Moreover, the ranks remain the same way regardless of any public disclosure.[5]

These signals are exogenous, and private. The regulator does not observe these signals. However, notice that the signals are about the fundamental at the beginning when the shock arrives. Unlike the agents, the regulator can check the the residual fundamental from time to time and disclose additional information via stress tests.

Endogenous public information (stress test results) Under a policy $\Gamma = (N, k)$, agent i in group n observes the first n stress test results $\zeta^n := \{\zeta_1, \zeta_2, \dots \zeta_n\}$. Let $\Omega^n := \{\mathsf{P}, \mathsf{F}\}^n$ be the set of all public information from the stress test that can be observed by group n agents.

Strategy, Solution Concept, and Adversarial Selection The strategy of agent i in the group n is a mapping $\rho_i : \mathbb{S} \times \Omega^n \to [0,1]$, whereby $\rho_i(s_i, \zeta^n)$ denotes the probability that an agent attacks. We adopt the *perfect Bayesian equilibrium (PBE) in monotone strategies* as our solution concept (henceforth referred to as equilibrium) for the dynamic coordination game played by the agents.

Definition 2 (Monotone Strategy) *A strategy ρ_i is said to be monotone if and only if*

1. *the agent is less likely to attack when her private signal is higher — i.e., $\rho_i(s_i', \zeta^n) \leq \rho_i(s_i, \zeta^n)$ for any $s_i' > s_i$ and any $\zeta^n \in \Omega^n$; and*

2. *the agent is less likely to attack when the bank passes a stress test rather than fails — i.e., $\rho_i(s_i, \zeta^{n-1} \cup \{\zeta_n = \mathsf{P}\}) \leq \rho_i(s_i, \zeta^{n-1} \cup \{\zeta_n = \mathsf{F}\})$ for any $s_i \in \mathbb{S}$ and $\zeta^{n-1} \in \Omega^{n-1}$.*

We will characterize the equilibrium in the next Section. Here, for expositional purpose, we introduce the necessary notations in an abstract way. Given a stress test policy Γ, let $\hat{\boldsymbol{\rho}}(\Gamma)$ be an equilibrium profile of strategies, and $\mathcal{P}_f(\Gamma, \hat{\boldsymbol{\rho}}(\Gamma))$ be the ex-ante probability of bank failure in this equilibrium. There are multiple equilibria. Let $\mathcal{E}(\Gamma)$ be the set of equilibria.

We consider adversarial selection. Under any policy Γ, we focus on the worst equilibrium, denoted by $\boldsymbol{\rho}^*(\Gamma)$, which has the highest probability of failure, that is, $\mathcal{P}_f(\Gamma, \boldsymbol{\rho}^*(\Gamma)) \geq \mathcal{P}_f(\Gamma, \hat{\boldsymbol{\rho}}(\Gamma))$ for all $\hat{\boldsymbol{\rho}}(\Gamma) \in \mathcal{E}(\Gamma)$. We define

$$\mathcal{P}_f(\Gamma) \equiv \mathcal{P}_f(\Gamma, \boldsymbol{\rho}^*(\Gamma)).$$

as the probability of failure under stress test policy Γ in the worst equilibrium.

[5]Formally, if $f(.)$ is log-concave, then for any $s^{''} > s^{'}$, $\frac{p(\theta|s^{''})}{p(\theta|s^{'})} = \frac{f((\theta - s^{''})/\rho)}{f((\theta - s^{'})/\rho)}$ is increasing in θ (assuming the denominator is positive). That is, the posterior beliefs are ranked according to likelihood ratio. This implies for any event A, $\frac{p(\theta|s^{''}, \theta \in A)}{p(\theta|s^{'}, \theta \in A)}$ is also increasing in θ (See Theorem 1.4.6. in Müller and Stoyan (2002)).

2.5 The Regulator's Problem

The regulator is an information designer, who before the game begins, commits to a stress test disclosure policy. If the shock arrives, then at the specified dates, the regulator conducts the tests and publicly discloses the pass/fail results. These tests reveal some partial information which influences the distribution of the posterior beliefs of agents, which in turn, affects their equilibrium strategies. It is important to note the following four features of this information design problem. First, we assume that the regulator does not know the private information the agents receive, and she only chooses a special type of disclosure policy called the stress tests, which is a dynamic, public, and binary disclosure. Second, the stress test policy we study here is an endogenous disclosure since the results from stress tests depend not only on the exogenous fundamental θ but also on the agents' choices w^t. Third, the regulator anticipate that for any stress test policy she designs, the agents will play the worst equilibrium in the resulting game (adversarial design). Finally, we assume that due to reputational concern, the regulator only chooses a stress test policy with no false-positives, that is, never gives a pass to a bank which, in the worst equilibrium, does not survive in the end.[6]

The regulator is a social planner who cares about the bank's payoff as well as the externality the bank imposes on the financial system when it fails. If the bank survives, it retains a charter value of $B > 0$; otherwise, if it fails, this value is forfeited. In addition, bank failure incurs a social cost of $\chi > 0$. Recall that the shock arrives with probability π. Therefore, for any given policy Γ and based on the worst-possible equilibrium under this policy, the bank fails with probability $\pi\mathcal{P}_f(\Gamma)$ and it will survive with probability $(1-\pi)+\pi(1-\mathcal{P}_f(\Gamma))$. We define the regulator's expected payoff for any given policy Γ as

$$\Lambda(\Gamma) \equiv \left((1-\pi)+\pi(1-\mathcal{P}_f(\Gamma)\right) B - \pi\mathcal{P}_f(\Gamma)\chi - C(\Gamma), \tag{3}$$

where $C(\Gamma)$ is the cost of conducting stress test Γ. We assume that the cost of running the stress tests $C(\Gamma)$ is not sensitive to the strength k but it is increasing and convex in the frequency N. Therefore, abusing the notation, we will write the cost as $C(N)$ and the "marginal" cost $\Delta C(N) \equiv C(N+1) - C(N)$, which is strictly positive and weakly increasing.

[6] If a bank that is issued a passing grade, fails, then it may shake the market's opinion of the regulator's competence in screening bad banks from good ones and makes future regulatory disclosures ineffective. In this sense, false-positive tests causes a tremendous reputational damage, and so the regulator avoids such policies at all costs. See the following excerpt from Hirtle and Lehnert (2015),

"Banks that show relatively high post-stress capital ratios would presumably be perceived as good risks by investors. The sudden collapse of such a bank, perhaps caused by an idiosyncratic event, or in the face of a macro stress less severe than the stress scenario, could shake confidence in the entire stress testing regime, calling into question not just the resiliency of other participating banks, but also of the competence of the supervisory authorities. Ultimately, confidence could be sufficiently diminished to precipitate a coordination failure among investors and a rush to pare exposures to the banking system."

2.6 Interpretations/Discussions of modeling choices

Sequential Moves: A crucial aspect of our model of dynamic coordination is the sequential nature of agents' moves. Such sequential moves naturally arise in practice. One example is the debt structure of a financial institution featuring (1) maturity mismatch and (2) asynchronous debt maturity dates. In Section 7, we provide such an example and analyze the stress test policy when the bank chooses the debt structure. Another practical interpretation is that a bank with maturity mismatch is sequentially approaching atomistic investors to fund its existing asset worth θ^0. The investors are privately informed regarding the bank's asset value and will invest \$1 if they do invest. If however, they decide not to invest, the bank must liquidate the unfunded proportion of the asset. Therefore, at any time $t \in [0, 1]$, the bank's fundamental or asset value is $\theta^t = \theta^0 - \int_0^t a_i di$.

Fundamental: The bank's fundamental θ can be broadly interpreted. For a bank facing an adverse liquidity distress event, we can interpret the fundamental as the liquidity position of the bank. This includes the amount of cash it can raise by borrowing against the long-term asset, the net value of High-Quality Liquid Assets (HQLA), or any emergency liquidity support the bank may have access to from other financial institutions or the regulator. This liquidity erodes as the bank fails to refinance its debts. Alternatively, θ may also be interpreted as the value of the assets relative to the bank's funding. When θ is thus interpreted, investors who short the bank's equity or creditors who decide not to lend reduce this net value of the bank's assets.

Stress Tests: The stress test disclosure policy may provide information regarding the bank's liquidity position as in liquidity stress tests, or the bank's asset values as in capital stress tests.

Liquidity Mismatch Index: The stress test policy proposed in this paper can be interpreted along the lines of the Liquidity Mismatch Index (LMI) in Bai, Krishnamurthy and Weymuller (2018) for conducting liquidity stress tests. The index is constructed under the assumption that in a liquidity distress event, the creditors will extract maximum liquidity and the bank will maximize the liquidity that it can raise. Thus, the LMI index captures the maximum assistance the bank may require. In our setting, the LMI index at time t is $\text{LMI}_t = (1 - t - \theta^t)$, i.e., the maximum withdrawal $(1 - t)$ from all the remaining agents, net of the liquid holdings (θ^t) of the bank. Recall that the bank passes the n-th stress test at $t = (n - 1)t_N$ iff the per-capita residual liquidity is sufficiently high, i.e., $\theta^t \geq k(1 - t)$, which is equivalent to $\text{LMI}_t \leq (1 - k)(1 - t)$. Put differently, passing a stress test at time t means LMI_t (the maximum assistance the bank may need at time t) is sufficiently low.

Capital Stress Test: The fundamental θ^t can be the profitability of the bank's assets at time t, after some of the creditors have made their decision, and the stress tests are capital stress tests as in Dodd-Frank Act Stress Tests (DFAST). The capital stress tests in essence disclose whether the bank has sufficient equity capital or funding to survive a shock to its asset values. If D_t is the amount owed to the remaining creditors, then the equity position $E_t = \theta^t - D_t$. Therefore in capital stress tests, the disclosure regarding equity ratios e.g. capital adequacy ratio

$$\frac{E_t}{\theta^t} > l \equiv \theta^t > \underbrace{\frac{1}{1-l}}_{=k} D_t$$

under shocks to the bank's assets profitability, can be mapped to the stress test policy in this paper. The creditors in this interpretation may also include long-term creditors, who are concerned with the profitability of the bank's assets as opposed to merely its liquidity position.

Agents' Prior: The investors' prior regarding θ has support $[\underline{\theta}, \overline{\theta}]$. For most part, we consider this as the premitive of the model. In Section 7 we show how other regulatory policies such as the Liquidity Coverage Ratio, may increase $\underline{\theta}$ and improve social welfare by enhancing the stability of the financial system.

Stress Test Strength: The strength of the stress test is akin to the stress scenarios for stress tests. A higher strength implies that the stress scenario is more severe and it will be more difficult for banks to pass such a stress test. Moreover, note that π does not directly affect the strength of the test for a given N, i.e. the severity of the stress scenario for a given frequency does not depend on the likelihood of the stress event. This is in line with the current methodology behind stress tests conducted by the Federal Reserve.[7]

3 A simple example

Before we establish the formal result, we resort to a simple example to illustrate the usefulness of frequent tests. Recall that under one-time stress test $(1, k)$, the game boils down to a static bank-run game (as in Goldstein and Huang (2016)). This is our benchmark setup. We will add one more test — policy $(2, k)$ and show how it increases the probability of survival in the worst case.

Let's start with the single test policy $(1, k)$. It is well known that the worst equilibrium is in cutoff strategies — attack if the bank fails the test, or if the bank passes the test but $s_i < \hat{s}$. This means there exists $k \leq \hat{\theta}$ such that the bank fails the test and does not survive if $\theta < k$; the bank

[7]See the following excerpt from Federal reserve disclosure regarding the Dodd-Frank Act Stress Tests, https://www.federalreserve.gov/newsevents/pressreleases/bcreg20180201a.htm *"The adverse and severely adverse scenarios describe hypothetical sets of events designed to assess the strength of banking organizations and their resilience. They are not forecasts."*

passes the test but does not survive if $\theta \in [k, \hat{\theta})$; the bank passes the test and survive if $\theta \geq \hat{\theta}$, where $\hat{\theta}$ is such that the aggregate attack at $\theta = \hat{\theta}$ is exactly $\hat{\theta}$:

$$F(\frac{\hat{s} - \hat{\theta}}{\sigma}) = \hat{\theta}$$

If the agents do not attack whenever the bank passes the test ($\hat{s} = k - \sigma/2$), then $\hat{\theta} = k$; otherwise, $\hat{\theta} > k$. Notice if all the agents ignore their private signals and perfectly coordinate their actions based on the stress test result, then an agent will do the same. Therefore, there is always an equilibrium with $\hat{\theta} = k$. If $\hat{\theta} > k$, then it must be that the marginal agent (with signal $\hat{s}$) is indifferent between attacking and not attacking — that is,

$$P(\theta \geq \hat{\theta} | \hat{s}, \theta \geq k) = p.$$

Notice that the LHS is the belief of the marginal agent ($\hat{s}$) that the bank will survive ($\theta \geq \hat{\theta}$) after learning that the bank has passed the test ($\theta \geq k$), and indifference requires that this belief is exactly p.

We are interested in the equilibrium with the highest $\hat{\theta}$. Suppose that θ^* is the worst equilibrium fundamental cutoff under policy $(1, k)$ and $\theta^* > k$. The reason θ^* is the worst equilibrium is that for any cutoff $\hat{\theta} > \theta^*$, the corresponding marginal agent (satisfying the first equality) believes that the probability of survival is strictly above p, and hence, strictly prefers not attacking.[8]

Next, consider policy $(2, k)$. Similar to $(1, k)$, the worst equilibrium will be as follows. An agent in group $n = 1, 2$ attacks if the bank fails any of the tests so far, or if it passes all the tests so far but $s_i < \hat{s}_n$. Given the cutoff strategies there exists $k \leq \underline{\theta}_1 \leq \hat{\theta}$ such that the bank fails the first test iff $\theta < k$. When the bank fails the first test, all the agents in group 1 attacks, which means the bank also fails the second test, and all the agents in group attack, and hence, the bank does not survive in the end. The bank passes the first test but fails the second test and does not survive iff $\theta \in [k, \underline{\theta}_1)$. The bank passes both tests but still does not survive in the end iff $\theta \in [\underline{\theta}_1, \hat{\theta})$, and the bank survives in the end iff $\theta \geq \hat{\theta}$.[9]

It is easy to see that as in the case of $(1, k)$ policy, there is an equilibrium with $\hat{s}_1 = \hat{s}_2 = k - \sigma/2$, which implies $\hat{\theta} = \underline{\theta}_1 = k$. That is, if all the agents ignore their private signals and perfectly coordinate their actions based on the stress test result, an agent will do the same. Consider an equilibrium with $\hat{\theta} > k$. Notice that when the agents in the second group see that the bank has passed both tests, they learn more than the agents in the first group ($\underline{\theta}_1 \geq k$). This makes the second group more optimistic than the first group regarding the probability of survival, and hence,

[8] Otherwise, there is $\hat{\theta} > \theta^*$ where the equality holds, which gives us a worse equilibrium.

[9] Notice that it is possible that the bank may pass the first test but fails the second test (because enough agents attack from group 1). However, in the worst case, the bank can never fail the first test but pass the next one (since all the agents from group 1 will attack).

for any θ such that the bank passes both tests ($\theta \geq \underline{\theta}_1$), there will be less attack from group 2 than from group 1. Therefore, for any $\hat{\theta}$, the marginal signals must be such that

$$\hat{s}_1 \geq \hat{s} \geq \hat{s}_2.$$

Moreover, if $\hat{\theta} > k$, then at least some agents from the first group will attack the bank even when it passes the first test. In this equilibrium, the marginal agent ($\hat{s}_1$) must be indifferent between attacking and not attacking — that is, $P(\theta \geq \hat{\theta}|\hat{s}_1, \theta \geq k) = p$.

Below, we provide a numerical example to illustrate that in the worst case, the bank is more likely to survive under policy $(2, k)$ than under policy $(1, k)$. Mayur will add an example with $(1, k)$ and $(2, k)$ and show that the worst eqm θ^* falls.

In fact, this above comparison hold true in general. Suppose θ^* is the worst equilibrium fundamental cutoff under $(1, k)$. We argue that there cannot be an equilibrium fundamental cutoff $\hat{\theta}$ under policy $(2, k)$ where $\hat{\theta} \geq \theta^*$. The argument follows two simple steps. First, for any $\hat{\theta} \geq \theta^*$, the corresponding marginal signal $\hat{s}_1$ of group 1 under $(2, k)$ policy must be strictly higher than the marginal signal $\hat{s}$ under $(1, k)$ policy (since more attack from group 1). Second, if $\hat{s}$ prefers not attacking (since θ^* is the worst equilibrium), then $\hat{s}_1$ strictly prefers not attacking, and hence $\hat{\theta}$ cannot be an equilibrium.

More formally, notice that when $\theta = k$ and $k < \theta^*$, the proportion of attack from group 1 is at least k. Therefore, when $\theta = k$, the bank will pass the first test but fail the second test ($\underline{\theta}_1 > k$). Accordingly, the second group is strictly more optimistic than the first group when they see that the bank passes both tests, and thus, for any for $\theta \geq \underline{\theta}_1$, the attack from the second group is strictly less than the attack from the first group. This implies that when $\theta = \hat{\theta}$ and $\hat{\theta} \geq \theta^* \geq \underline{\theta}_1 > k$, the proportion of attack from group 1 must be strictly higher than $\hat{\theta}$. Therefore, the corresponding marginal agent in group 1 ($\hat{s}_1$) must be such that

$$F(\frac{\hat{s}_1 - \hat{\theta}}{\sigma}) > \hat{\theta}.$$

Recall the above relation holds with equality under policy $(1, k)$, which means that for any fundamental cutoff $\hat{\theta} \geq \theta^*$, the corresponding marginal signals are such that $\hat{s}_1 > \hat{s} > \hat{s}_2$ (Step 1). Accordingly,

$$P(\theta \geq \hat{\theta}|\hat{s}_1, \theta \geq k) > P(\theta \geq \hat{\theta}|\hat{s}, \theta \geq k) \geq p.$$

The last inequality follows since θ^* is the worst equilibrium under $(1, k)$ and $\hat{\theta} \geq \theta^*$, and the first inequality follows since $\hat{s}_1 > \hat{s}$. Therefore, $\hat{s}_1$ strictly prefers not attacking (Step 2), and hence, the fundamental equilibrium cutoff under $(2, k)$ must be strictly lower than θ^*.

Goldstein and Huang (2016) shows that if k is sufficiently high ($k > k^*$), then $P(\theta \geq \theta^*|s^*, \theta \geq k)$

is always above p. This means there is no equilibrium with $\theta^* > k$. Put differently, if $k > k^*$, then even in the worst case, the agents ignore their private signals and perfectly coordinate their actions based on the stress test result. We will argue that as the regulator conducts more tests, k^* falls.

4 Equilibrium

4.1 Cutoff Equilibrium

In this section, we study the agents' problem given a stress test policy Γ. Recall that the agents are privately informed and they play a dynamic coordination game. Our equilibrium concept is PBE where the agents play monotone strategies. Under monotone strategies, an agent is less likely to attack if her private signal is higher (ρ_i is decreasing in s_i), and if the bank passes a test $n' \leq n$ rather than fails. Therefore, if the bank survives under θ', then it will also survive under $\theta'' > \theta'$. This gives us a threshold θ^* such that the bank survives iff $\theta \geq \theta^*$. We focus on the worst equilibrium (θ^* higher than all other equilibria).

Lemma 1 *The worst monotone PBE must have the following two properties. (1) if ζ^n is such that $\zeta_{n'} = 0$ for some $n' \leq n$, then $\rho_i(s_i, \zeta^n) = 1$ for all $s_i \in \mathbb{S}$. (2) if ζ^n is such that $\zeta_{n'} = 1$ for all $n' \leq n$ then $\rho_i(s_i, \zeta^n) = \mathbb{1}\{s_i < s_n^*\}$.*

This result straightforward for a one-time stress test policy. The above lemma only extends the argument for the general stress test policy. See the appendix for the formal proof. Let $\tilde{\mathcal{E}}(\Gamma)$ be the set of equilibrium that satisfies the above two properties. Since we are interested in the worst equilibrium $\boldsymbol{\rho}^*(\Gamma)$, and $\boldsymbol{\rho}^*(\Gamma) \in \tilde{\mathcal{E}}(\Gamma) \subseteq \mathcal{E}(\Gamma)$, it is without loss of generality to discard the equilibrium in $\mathcal{E}(\Gamma) \setminus \tilde{\mathcal{E}}(\Gamma)$. Henceforth, we only focus on equilibrium in $\tilde{\mathcal{E}}(\Gamma)$. Notice that an equilibrium in $\tilde{\mathcal{E}}(\Gamma)$ can be described simply by N cutoffs $\boldsymbol{s}^* := \{s_n^*\}_{n=1}^N$. Abusing notation, henceforth, we refer to an equilibrium as $\boldsymbol{s}^*$ rather than $\boldsymbol{\rho}^*$. Given $\mathbf{s}^*$, there exists θ^* such that the bank survives in the end iff $\theta \geq \theta^*$. $\hat{\mathbf{s}} \in \tilde{\mathcal{E}}(\Gamma)$ is the worst equilibrium under stress test policy Γ if $\theta^*(\hat{\mathbf{s}}, .) \geq \theta^*(\mathbf{s}^*, .)$ for all $\mathbf{s}^* \in \tilde{\mathcal{E}}(\Gamma)$. We define $\hat{\theta}(\Gamma) \equiv \theta^*(\hat{\mathbf{s}}, \Gamma)$ and

$$\mathcal{P}_f(\Gamma) \equiv \mathbb{P}(\theta < \hat{\theta}(\Gamma)) = \frac{\hat{\theta}(\Gamma) - \underline{\theta}}{\bar{\theta} - \underline{\theta}}. \tag{4}$$

Thus, under the stress test policy Γ, in the worst equilibrium, the regime survives iff $\theta \geq \hat{\theta}(\Gamma)$ and the ex-ante probability that the bank survives is $(1 - \mathcal{P}_f(\Gamma))$.

4.2 Characterization of Equilibrium Cutoffs

For any $\mathbf{s}^*$, there exists a weakly increasing sequence

$$k = \underline{\theta}_0 \leq \underline{\theta}_1 \leq \ldots \leq \underline{\theta}_{n-1} \leq \underline{\theta}_N = \theta^*$$

such that the bank passes the first n tests iff $\theta \geq \underline{\theta}_{n-1}$, and survives in the end iff $\theta \geq \theta^*$.

Definition 3 (Persuading group n) *Under the stress test policy Γ and an equilibrium $\boldsymbol{s}^*(\Gamma) \in \tilde{\mathcal{E}}(\Gamma)$, group n is persuaded if, in this equilibrium, the agents in the group n never attacks if the bank passes the first n stress tests, that is, $s_n^* = \underline{\theta}_{n-1} - \sigma/2$.*

Suppose that under stress test policy $\Gamma = (N, k)$, there is an equilibrium $\mathbf{s}^*(\Gamma) \in \tilde{\mathcal{E}}(\Gamma)$ such that some group $n = 1, 2, \ldots, N$ is not persuaded. This means the policy Γ cannot be robustly persuasive (See definition 4).

If under equilibrium $\boldsymbol{s}^*$, group n is persuaded, then by definition, $s_n^* = \underline{\theta}_{n-1} - \sigma/2$. Since, an agent with signal $s_n^* = \underline{\theta}_{n-1} - \sigma/2$ does not believe that θ can be higher than $\underline{\theta}_{n-1}$, the only way she is persuaded is if $\theta^* = \underline{\theta}_{n-1}$. This means once the bank passes the nth test, it will pass all the subsequent tests and survive in the end. Therefore, for any $\mathbf{s}^*$, there exists $\overline{n} \in \{1, 2, \ldots, n, N+1\}$ such that all the groups starting from $\overline{n}$ are persuaded. If no group is persuaded, we use the convention that $\overline{n} = N + 1$.

If under $\mathbf{s}^*$ group n is not persuaded, then an agent i in group n wants to attack iff $s_i < s_n^*$. Notice that she believes that the bank will survive with probability

$$P(\theta \geq \theta^* | s_i, \theta \geq \underline{\theta}_{n-1}) = \frac{F\left(\frac{s_i - \theta^*}{\sigma}\right)}{F\left(\frac{s_i - \underline{\theta}_{n-1}}{\sigma}\right)}.$$

It follows from Assumption 1 that this belief is increasing in s_i. If group n is not persuaded then it must be that the marginal agent s_n^* believes that the probability of survival is p.

Consider a non-persuaded group $n < \overline{n}$. Note that since $\underline{\theta}_n$ is weakly increasing, s_n^* is weakly decreasing in n (follows from the indifference condition and Assumption 1). That is, there is less attack from later groups. If a group n is not persuaded, then when $\theta = \underline{\theta}_{n-1}$, the proportion of attack from group n must be strictly above k. Otherwise, since there are even fewer attacks later, once the bank passes the n-th test, it will pass all the subsequent tests and survive in the end, that is, $\underline{\theta}_{n-1} = \theta^*$. This contradicts the fact that group n is not persuaded. This implies that as long as group n is not persuaded, the $(n+1)$-th test reveals new information $\underline{\theta}_n > \underline{\theta}_{n-1}$, and when $\theta = \underline{\theta}_n$, the size of attack from the first n group leaves k per-capita residual fundamental.

Proposition 1 (Equilibrium) *Given any stress test policy Γ, $\mathbf{s}^* = \{s_n^*\}_{n=1}^N$ constitutes an equilibrium if there exists $\overline{n}(\mathbf{s}^*) \in \{1, \ldots, n, N+1\}$ such that all groups $n \geq \overline{n}$ are persuaded, that is,*

$s_n^* = \underline{\theta}_{\overline{n}-1} - \frac{\sigma}{2}$, *and for any non-persuaded group* $n \leq \overline{n} - 1$, s_n^* *is such that the marginal agent is indifferent between attacking and not attacking*

$$\frac{F\left(\frac{s_n^* - \theta^*}{\sigma}\right)}{F\left(\frac{s_n^* - \underline{\theta}_{n-1}}{\sigma}\right)} = p, \tag{I_n}$$

where the threshold for passing the n*-th test* $\underline{\theta}_{n-1}$ *is such that*

$$\underline{\theta}_{n-1} - \frac{1}{N} \sum_{n'=1}^{n-1} F\left(\frac{s_{n'}^* - \underline{\theta}_{n-1}}{\sigma}\right) = k\left(1 - \frac{n-1}{N}\right) \tag{ζ_n}$$

and the bank survives in the end if

$$\theta^* = \underline{\theta}_{\overline{n}-1}. \tag{S}$$

The formal proof is relegated to the appendix. The result follows from the argument preceding the proposition. Notice that when $\theta = \underline{\theta}_{n-1}$ the attack from the first $(n-1)$ groups leaves k per-capita residual fundamental (See equation (ζ_n)). Thus, when the bank passes the n-th test, the agents in group n learn that $\theta \geq \underline{\theta}_{n-1}$. If they are not persuaded, then marginal agent s_n^* must believe that the probability that the bank will survive is p (See equation (I_n)). If they are persuaded, then all the subsequent groups must be persuaded, that is, $\overline{n} = n$, and $\theta^* = \underline{\theta}_{n-1}$ (See equation (S)). If no group is persuaded then we use the convention that $\theta^* = \underline{\theta}_N$.

Under the policy $\Gamma = (N, k)$, if there exists equilibrium $\mathbf{s}^* \in \mathcal{E}(\Gamma)$ such that $\overline{n}(\mathbf{s}^*) = 1$, then under this equilibrium, all the agent in all the groups disregard their private signal s_i and coordinate on the stress test results. This equilibrium is referred to as *obedient equilibrium*. The next corollary states that, under any stress test policy Γ, there is always an obedient equilibrium.

Corollary 1 (Obedient Equilibrium) *Under any policy* $\Gamma = (N, k)$, $\mathbf{s}^{o*}(\Gamma) \equiv \{s_n^{o*} = k - \frac{\sigma}{2}\}_{n=1}^N \in \tilde{\mathcal{E}}(\Gamma)$. *In this equilibrium,* $\overline{n}(\mathbf{s}^{o*}) = 1$ *and* $\theta^*(\mathbf{s}^{o*}, \Gamma) = k$.

It is easy to see that if all the agents follow the strategy $\mathbf{s}^{o*}$, then whenever $\theta \geq k$, the bank will pass the first test, and then all the subsequent tests and survive in the end. Therefore, any agent i in any group n, regardless of his private signal, prefers not attacking when the bank passes all the tests so far, which makes it an equilibrium.[10]

If the regulator is optimistic that when she chooses a policy Γ, the agents will always play this obedient equilibrium, then she can simply choose $\Gamma = (1, 0)$, and a solvent bank will never fail. However, there can be many equilibria other than the obedient one. We refer to such as equilib-

[10]Notice that passing a test eliminates the lower dominance region, and makes it possible that all the agents coordinate on the publicly disclosed information. This is a well-known result from a static setting, and we show here that the same result carries over to our dynamic setting.

rium as disobedient equilibrium. We follow the adversarial design approach where the regulator anticipates that the agents will play the worst equilibrium with the highest fundamental cutoff $\hat{\theta}(\Gamma)$.

5 Main Results

Our main objective is to identify the stress test policies that are *robustly persuasive*.

Definition 4 (Robustly Persuasive Stress Test) *A stress test policy $\Gamma = (N, k)$ is robustly persuasive if, under the worst equilibrium $\boldsymbol{\sigma}^*(\Gamma)$, the agents ignore their private signals and coordinate their actions based on the stress test results. That is,*

$$\mathbf{\Gamma}^P := \{\Gamma | \hat{\boldsymbol{s}}(\Gamma) = \boldsymbol{s}^{o*}(\Gamma)\}. \tag{$\mathbf{\Gamma}^P$}$$

Since a disobedient equilibrium will have higher probability of failure, under a robustly persuasive Γ, it must be that there is no disobedient equilibrium in $\tilde{\mathcal{E}}(\Gamma)$. By Corollary 1, when $\Gamma \in \mathbf{\Gamma}^P$, $\hat{\theta}(\Gamma) = k$ and $\mathcal{P}_f(\Gamma) = \mathbb{P}(\theta < k)$. If $\theta \geq k$, the bank passes all the tests, and no agent attacks, and the bank survives; and if $\theta < k$ the bank fails all the tests, and all the agents attack, and the bank fails eventually. Below, we characterize all robustly persuasive stress tests policies, and then show the trade-off between frequency and strength of stress tests.

5.1 Robustly Persuasive Stress Test

Theorem 1 *For any N, a stress tests policy $\Gamma = (N, k)$ is robustly persuasive iff $k > k^*(N)$, where*

$$k^*(N) := \inf \left\{ k \in [0,1] : \min_{x \in [k,1]} \frac{x}{F\left(F^{-1}(x) + \frac{x-k}{N\sigma}\right)} \geq p \right\}. \tag{5}$$

Thus, the set of robustly persuasive stress tests $\mathbf{\Gamma}^P = \{(N,k) | N \in \mathbb{Z}_+, k > k^*(N)\}$.

Proof. Consider any policy $\Gamma = (N, k)$ that satisfies condition $k > k^*(N)$ as defined in (5). Suppose this policy is not robustly persuasive; that is, there exists some equilibrium $\mathbf{s}^*$ such that $\overline{n}(\mathbf{s}^*) > 1$. Under such equilibrium, the first $\overline{n} - 1$ groups are not persuaded, while starting from group $\overline{n}$ all the subsequent groups are persuased, that is, $\theta^* = \underline{\theta}_{\overline{n}-1}$ (see Proposition 1). Below, we show that such equilibrium cannot exist given $k > k^*(N)$.

Recall from Proposition 1 that under equilibrium $\mathbf{s}^*$, the bank passes the n-th test iff $\theta \geq \underline{\theta}_{n-1}$, where $\underline{\theta}_{n-1}$ is as defined in equation (ζ_n). Rearranging this equation we get

$$\underline{\theta}_{n-1} - k = \frac{1}{N} \sum_{n'=1}^{n-1} \left(F\left(\frac{s^*_{n'} - \underline{\theta}_{n-1}}{\sigma} \right) - k \right).$$

The LHS is the excess strength required to pass the n-th test compared to passing the first test, while the RHS is the excess attack (in excess of k) from the first group $(n-1)$ groups when $\theta = \underline{\theta}_{n-1}$. Taking the difference for two consecutive tests (n) and $(n-1)$, we get

$$\underline{\theta}_{n-1} - \underline{\theta}_{n-2} = (\underline{\theta}_{n-1} - k) - (\underline{\theta}_{n-2} - k) \leq \frac{1}{N}\left(F\left(\frac{s^*_{n-1} - \underline{\theta}_{n-1}}{\sigma}\right) - k\right) \tag{6}$$

The inequality follows because $\underline{\theta}_{n-2} < \underline{\theta}_{n-1}$, which means the excess attack from the first $n-2$ groups is higher when $\theta = \underline{\theta}_{n-2}$ than when $\theta = \underline{\theta}_{n-1}$. The equality holds when $n = 2$. This is because when $n = 2$, there is no group moving before the first test.

Consider group $\overline{n} - 1$, the last group that is not persuaded. The marginal agent $s^*_{\overline{n}-1}$ must believe that the bank will survive with probability p (See equation (I_n)), that is,

$$\frac{F\left(\frac{s^*_{\overline{n}-1} - \theta^*}{\sigma}\right)}{F\left(\frac{s^*_{\overline{n}-1} - \underline{\theta}_{\overline{n}-2}}{\sigma}\right)} = \frac{F\left(\frac{s^*_{\overline{n}-1} - \theta^*}{\sigma}\right)}{F\left(\frac{s^*_{\overline{n}-1} - \theta^*}{\sigma} + \frac{\theta^* - \underline{\theta}_{\overline{n}-2}}{\sigma}\right)} = p. \tag{7}$$

Since this is the last non persuaded group, if the bank passes the next test, it will survive in the end — that is, $\theta^* = \underline{\theta}_{\overline{n}-1}$. Let us define

$$x := F\left(\frac{s^*_{\overline{n}-1} - \theta^*}{\sigma}\right)$$

as the proportion of attack from the last non persuaded group $(\overline{n} - 1)$ when $\theta = \underline{\theta}_{\overline{n}-1} = \theta^*$. We can see from equation (6) that $x > k$ and

$$\theta^* - \underline{\theta}_{\overline{n}-2} \leq \frac{x - k}{N}. \tag{8}$$

Substituting this inequality (8) in (7), we get that the marginal agent $s^*_{\overline{n}-1}$ believes that the bank will survive with probability at least

$$\frac{x}{F\left(F^{-1}(x) + \frac{x-k}{N\sigma}\right)} =: G(x, N, k). \tag{G}$$

Thus, under a stress test policy $\Gamma = (N, k)$, for any equilibrium $\mathbf{s}^*$, if under $\theta = \theta^*$, $x \in (k, 1]$ is the proportion of attack in the last group that is not persuaded, then $G(x, N, k)$ is the lower bound of the belief of the marginal agent in the last group that is not persuaded. Recall that (8) holds with equality when $\overline{n} = 2$. In this case, the marginal agent's belief in the group 1 (the last non persuaded group) is exactly $G(.)$. If $k > k^*(N)$, then given the definition of $k^*(N)$ (See equation (5)),

$$G(x, N, k) > p.$$

Therefore, for any $x \in (k, 1]$ the marginal agent in group $\overline{n} - 1$ cannot be indifferent as required in equilibrium (See (I_n)). Therefore, when $k > k^*(N)$, there is no equilibrium $\mathbf{s}^*$ such that $\overline{n}(\mathbf{s}^*) \neq 1$. In other words, under any policy $\Gamma = (N, k)$ such that $k > k^*(N)$, the unique equilibrium is $\mathbf{s}_o^*(N, k)$ and, therefore, $\Gamma \in \mathbf{\Gamma}^P$.

On the other hand, if $k \leq k^*(N)$, then there exists $x \in (k, 1]$ such that $G(x, N, k) = p$. One can construct a disobedient equilibrium where the first group is not persuaded but the subsequent groups are persuaded. The equilibrium s_1^* and the resulting θ^* are such that when $\theta = \theta^*$, the proportion of attack from the first group $F(\frac{s_1^* - \theta^*}{\sigma}) = x$, and the residual strength is just enough to pass the second stress test $\theta^* - \frac{x}{N} = k(1 - \frac{1}{N})$ (and then all the subsequent stress tests). Together these imply the set of robustly persuasive tests $\mathbf{\Gamma}^P = \{(N, k) | N \in \mathbb{Z}_+, k > k^*(N)\}$.

■

Notice that the stress test policy $(N, k^*(N))$ is not robustly persuasive. However, (N, k) with $k > k^*(N)$ is robustly persuasive. This creates a technical problem because there is no minimum k that makes (N, k) robustly persuasive. Following the literature (See, for instance, Goldstein and Huang (2016)), we assume that the smallest feasible strength of stress test the regulator can choose that is above $k^*(N)$ is $k^*(N)^+ = k^*(N) + \epsilon$ for some small $\epsilon > 0$. In other words, there is no feasible $k \in (k^*(N), k^*(N)^+)$.

5.2 Trade off between strength and frequency

The optimal design of stress tests crucially depends on understanding the trade-off between strength and frequency. A stress test strength k may fail to persuade the agents when there are N tests. However, it may become robustly persuasive if there are more tests. In this section, we investigate the properties of the required strength for robustly persuasive policy $k^*(N)$.

Theorem 2 *The function $k^*(N)$ satisfies the following properties*

1. *There exists $N_0 \geq 1$ such that $k^*(N) = 0$ for all $N \geq N_0$.*

2. *$k^*(N)$ is strictly decreasing in N for $N \leq N_0 - 1$.*

3. *$k^*(N)$ is discretely convex in N; that is, $k^*(N) - k^*(N+1)$ is strictly decreasing in N for $N \leq N_0 - 1$.*

Accordingly, the set of robustly persuasive stress test $\mathbf{\Gamma}^P$ is convex.

The result follows from the fact that $G(x, N, k)$ is increasing and quasi-concave in (N, k). The formal proof is relegated to the appendix. Theorem 1 shows that for any given frequency of stress tests N, there exists a threshold strength $k^*(N)$ that makes the tests robustly persuasive; and Theorem 2 demonstrates how the required strength for robustly persuasive policy $k^*(N)$ changes with N. These results nest the findings of Goldstein and Huang (2016) and Basak and Zhou (2020). Goldstein and Huang (2016) consider the case of static coordination, i.e., $N = 1$, and find the lower bound for one-time robustly persuasive policy $k^*(1)$. Basak and Zhou (2020) demonstrate that, when the debt structure is sufficiently asynchronous ($N \geq N_0$), the continued viability of the borrower is sufficient to persuade all agents to coordinate on rolling over the maturing debt; that is, $k^*(N) = 0$ for all $N \geq N_0$ (the first part of Theorem 2). For those cases, there will be no run on any fundamentally sound banks and, therefore, the efficient coordination is achieved.

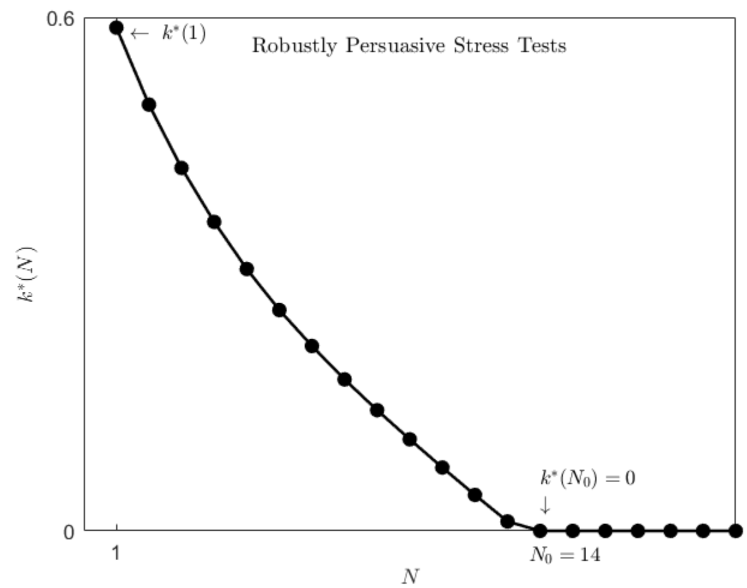
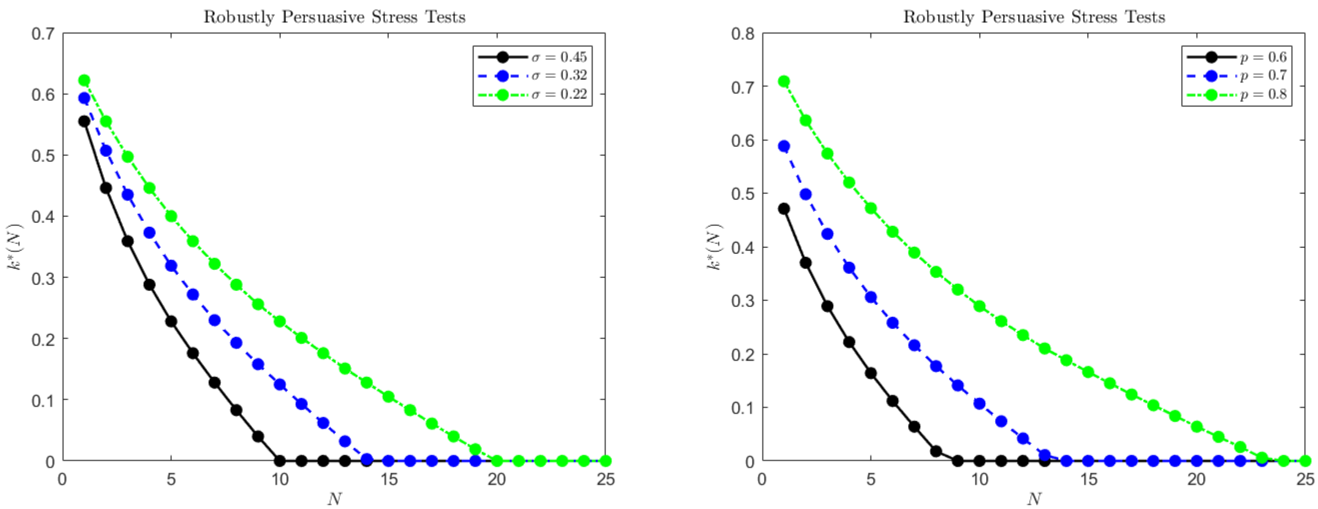


Figure 1: $k^*(N)$ - The strength required to make the n times repeated stress tests persuasive. The density $f = (2 + 4x) \cdot \mathbb{1}(-0.5 \leq x < 0) + (2 - 4x) \cdot \mathbb{1}(0 \leq x \leq 0.5)$ with $(p, \sigma) = (0.7, 0.33)$.

This paper provides a complete picture of how the required strength of a robustly persuasive stress test, k^*, depends on the its frequency, N. The second part of Theorem 2 demonstrates that more frequent stress tests are beneficial because it enables more information transmission across agents that facilitates coordination. More frequent tests means the strength for each test can be reduced while its persuasiveness maintains. This result follows from the fact that under higher N, when the future groups are persuaded, the excess strength needed to pass the next test is lower, making the agents more optimistic about the survival of the regime. Finally, the third part of Theorem 2 shows that the reduction in the required strength $k^*(N)$ falls as we increase the frequency of tests N. This means the marginal benefit of increasing the frequency of stress tests is decreasing. Figure 1 presents a graphical illustration of $k^*(N)$ for some specific parameters.

Next, we investigate how the model primitives affect the required strength $k^*(N)$ for robust persuasion. The two main model primitives are p — the agents reluctance to rollover, and σ — the scaling parameter for noisy private signals.

Proposition 2 *Given any frequency N, the required strength for robustly persuasive stress tests, $k^*(N)$, is non-decreasing in p and non-increasing in σ.*



(a) (b)

Figure 2: $k^*(N)$ - The strength required to make the N times repeated stress tests persuasive

The formal proof is relegated to the appendix. The Figure 2 below illustrates this result. Proposition 2 shows that it is easier to robustly persuade the agents if they are more willing to roll over (lower p), and if they are more uncertain about the bank's liquidity position (higher σ). To see this, consider $G(.)$, the lower bound on the belief of the marginal agent in the last non persuaded group (See equation (G)). Recall that required strength $k^*(N)$ for robust persuasion makes $G(.)$ higher than p. It is easier to achieve this if (1) p is lower and (2) σ is higher which makes $G(.)$ higher. This means the stress tests do not need to be as tough to be robustly persuasive, which gives us the above result.

5.3 Optimal Stress Test

The regulator does not see the private signals the agents have received. She commits to a stress test policy Γ. She anticipates that the agents will play the worst equilibrium, and due to reputational concern, she only chooses a stress test which does not generate false-positive results. It is easy to see that a stress test policy Γ does not generate false-positive results in the worst equilibrium iff Γ is robustly persuasive ($\Gamma \in \mathbf{\Gamma}^P$).[11]

[11] The argument is simple. If Γ is robustly persuasive, then in the worst equilibrium, once the bank passes the first test, no agent attacks, and so the bank passes the next test, and the next one, and so on. Therefore, a robustly

Therefore, the regulator's problem is to choose a stress test policy $\Gamma \in \mathbf{\Gamma}^P$ to maximize the social welfare

$$\Lambda(\Gamma) = ((1-\pi) + \pi(1-\mathcal{P}_f(\Gamma))\,B - \pi\mathcal{P}_f(\Gamma)\chi - C(\Gamma).$$

Recall that under a robustly persuasive stress test $\Gamma = (N, k)$, the probability of failure in the worst equilibrium is $\mathcal{P}_f = \frac{k-\underline{\theta}}{\overline{\theta}-\underline{\theta}}$. Thus, for any N, the regulator prefers the weakest test provided the tests are robustly persuasive ($k^*(N)^+$). Abusing notation, we write $\Lambda(N) = \Lambda(N, k^*(N)^+)$. If the regulator conducts more tests, then as shown in Theorem 2, she can lower the required strength $k^*(N)$ to be robustly persuasive. However, conducting more tests is costly. Since, $k^*(N)$ is convex (by Theorem 2) and $C(N)$ is convex (by assumption), the regulator's payoff $\Lambda(N)$ is concave in N, which gives a unique optimal stress test.

Proposition 3 *The regulator optimally chooses the stress test policy $\Gamma^s = (N^s, k^*(N^s)^+)$ and commits to it, where*

$$N^s = \min\left\{ N \in \mathbb{Z}^+ : k^*(N) - k^*(N+1) \leq \frac{\Delta C(N)(\overline{\theta}-\underline{\theta})}{\pi(B+\chi)} \right\}.$$

If the shock arrives, in the worst equilibrium, the agents ignore their private signals and perfectly coordinate their actions based on the stress test results, that is, $s_n^* = k - \frac{\sigma}{2}$ for all $n = 1, 2 \ldots N^s$. Thus, a bank with fundamental $\theta < k$ fails all the tests, and all the agents attack, and the bank does not survive in the end; while a bank with fundamental $\theta \geq k$ passes all the tests and no agent attacks, and so, it survives in the end.

The marginal benefit of increasing N comes from the reduction in the ex-ante probability of bank failure conditional on the arrival of liquidity shock; that is,

$$\mathcal{P}_f(N, k^*(N)) - \mathcal{P}_f(N+1, k^*(N+1)) = \frac{1}{\overline{\theta}-\underline{\theta}}\left(k^*(N) - k^*(N+1)\right)$$

The expected marginal benefit from choosing more tests is this reduction in the probability of failure times the charter value B, plus the externality χ, conditional on the arrival of liquidity distress (probability π); that is, $\frac{\pi(B+\chi)}{\overline{\theta}-\underline{\theta}}(k^*(N) - k^*(N+1))$. Since $k^*(N) - k^*(N+1)$ is decreasing (Theorem 2), this expected marginal benefit is decreasing in N. The above result then follows form equating the marginal benefit and marginal cost of increasing the frequency N.

persuasive stress test does not generate false-positives. On the other hand, if there is a false positive test, then it must be that in the worst equilibrium, some agents attack even after the bank passes all the tests so far, which means the stress test policy is not robustly persuasive.

6 Policy Implications

In this section, we relate our dynamic information disclosure policy to the current practice of stress tests conducted for financial institutions facing maturity mismatch in their balance sheets. We discuss: (1) how the frequency of stress tests can be used as an important policy lever, and (2) how the socially optimal frequency depends on macroeconomic conditions, and bank-specific features.

6.1 Frequency: A Key Policy Lever

A crucial message of this paper is that it is not just the severity of the stress tests but the ability to conduct such stress tests more often that helps the regulator in preventing runs. In practice we observe heterogeneity in the execution of stress test policy with differences in frequency of stress tests across financial institutions within a jurisdiction, and across geographies. Stress test policies with discretionary frequency are also being employed in the context of financial institutions other than bank holding companies, that engage in maturity transformation. For example, the European Securities and Market Authority (ESMA) requires fund managers to conduct liquidity stress tests for their funds with the minimum required annual frequency, however, the managers may exercise discretion and conduct more or less frequent stress tests depending on the market conditions.[12]

In our theoretical analysis, we demonstrate that a commitment to conduct frequent stress tests makes it easier to persuade agents to roll over their investment. that when the regulator commits to conducting and disclosing results from stress tests at a greater frequency, it becomes easier to persuade the agents to roll over their debt claims. The intuition is that conditional on future stress tests being persuasive, it becomes easier to persuade the current group of agents when the stress tests are conducted more frequently. The lawmakers in the US and the EU have both enacted laws that commit them to conducting frequent stress tests of financial institutions, e.g. The Dodd-Frank Act in the US and the Single Supervisory Mechanism (SSM) in the EU. However, such a commitment did not exist in the early stress test exercises. Our theory provides an additional perspective on how the commitment to conduct frequent stress tests possibly led to differential outcomes in early stress test exercises conducted in these jurisdictions. The success of the the 2009 Supervisory Capital Assessment Program (SCAP) stress test conducted in the US vis-a-vis the failure of the EU-wide stress tests conducted in 2011 in restoring market confidence in the banking system is attributed to governance aspects of the US tests including a credible and well-funded backstop facility, and the failure of the EU stress test to adequately incorporate the sovereign risk scenarios (tests not tough enough) Faria-e Castro, Martinez and Philippon (2017). However, another possible factor contributing to the success of the SCAP is that the US had already committed by law to conduct future regular stress tests, whereas Europe had not. That is, the market participants may have had

[12] See discussion on the frequency of stress tests: https://www.esma.europa.eu/sites/default/files/library/esma34-39-897_guidelines_on_liquidity_stress_testing_in_ucits_and_aifs_en.pdf

concerns about whether the European banks would survive despite the success of the current stress test. [13]

Why frequency matters: It is clear that the frequency of stress tests is an important dimension of policy design, but how does it help the policy makers? Consider the following scenario: The regulator is constrained to run stress tests at a pre-specified frequency N^s. Suppose the stress tests need to accommodate a more severe scenario that incorporates deterioration of market conditions. This is captured in our model through a higher p (or low σ or both), i.e. it is more difficult to convince the agents to rollover their debt. Conventional wisdom suggests that the regulator must run stress tests with higher strength. This is in line with Proposition (2) where we show that for a given N, under a higher p (low σ), the required strength of the stress test is higher. A more severe stress test increases the ex-ante likelihood of bank failure under distress. However, as per the Proposition (3), the optimal frequency and the corresponding strength of a robustly persuasive stress test are jointly determined. In other words, the regulator may be able to mitigate the higher likelihood of failure, if she could conduct more frequent tests. We illustrate the benefit of flexibility in choosing the frequency and strength of stress tests when the market conditions worsen in Figure (3). The regulator conducts stress tests at frequency N^s under the current market conditions. When the market conditions change from (p, σ) to (p', σ'), a regulator running stress tests at the same frequency N^s, must conduct tougher tests. However, a regulator who optimally chooses the frequency ends up choosing a much higher frequency $N^{s'}$ and lower strength that increases the likelihood of bank survival.

6.2 Determinants of Optimal Frequency

The regulators conducting stress tests must take into account the specific features of the financial institution while designing the stress test policy. Should the regulator mandate bank-specific stress test policies, taking into account bank-specific features such as, its systemic importance, or its balance sheet composition? In this subsection, we attempt to provide an answer to these questions. This will also give us some insight into what factors may be important for the choice of frequency of stress tests. For example, in its guidelines for stress tests of funds, ESMA allows discretion on

[13] See the excerpt below from Abramovich (2011):

"One future test has been announced by the EBA for the first half of 2011. The United States, on the other hand, has enacted comprehensive legislation requiring stress testing of banks by regulators as well as self- testing. Although the United States legislation has not yet been implemented, the statute offers a regulatory framework for future stress testing. The United States has a legislative mandate with definite regularity in its application."

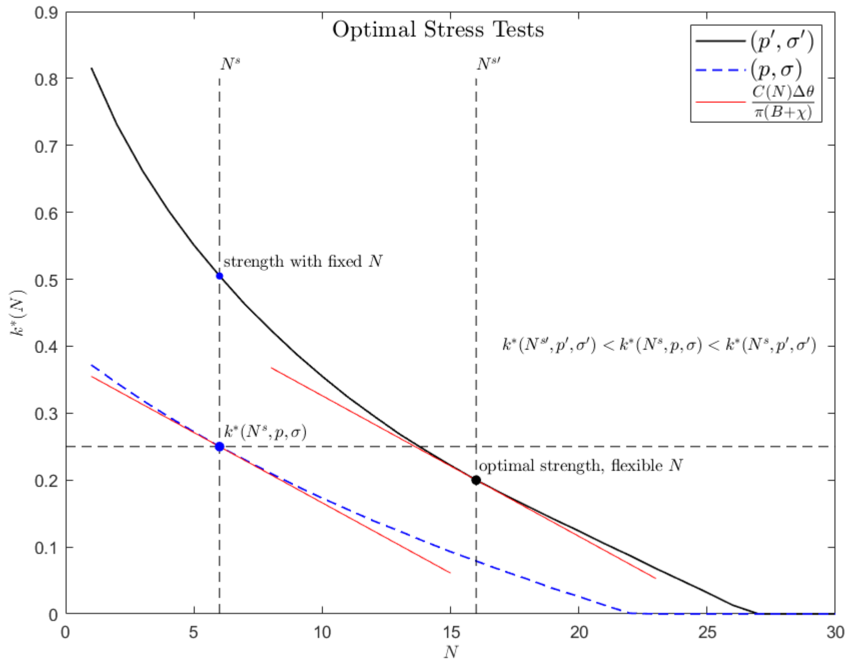


Figure 3: Optimal frequency of robustly persuasive stress tests when market conditions change.

the frequency of stress tests policy depending on the fund-specific features.[14]

Bank Specific Factors: Susceptibility to Macroeconomic Shocks

Observation 1 *N^s is increasing in π.*

Off-balance sheet exposures and a significant proportion of short-term wholesale funding may make a financial institution more susceptible to exogenous shocks due to worsening macroeconomic conditions. We capture this vulnerability of the financial institution through the parameter π. From Proposition (3) we can infer that N^s is increasing in π. For financial institutions more susceptible to adverse shocks, the frequency of stress tests should be higher as the expected marginal benefit from more tests is higher. In the guidelines for conducting stress tests for Alternative Investment Funds (AIFs), the European Security Market Authority (ESMA) acknowledges that the respondents (asset management firms) called for a higher frequency of stress tests when there is an emerging/imminent

[14] See bullet number 26 in the guidelines for stress testing document https://www.esma.europa.eu/document/guidelines-liquidity-stress-testing-in-ucits-and-aifs
When deciding on the appropriate frequency, managers should take into account the following: (a) the liquidity of the fund determined by the manager and any change in the liquidity of assets; (b) the frequency should be adapted to the fund rather than a 'one-size-fits-all' approach being taken to all funds operated by the manager; and (c) the nature of the vehicle (closed versus open ended), the redemption policy and LMTs, such as gates or side pockets, may be additional factors to take into consideration when determining the appropriate frequency of LST.

risk to fund liquidity.[15] This maps to our discussion, that when the probability of liquidity distress is higher, the frequency must be higher. In this case, the higher frequency allows for a less stringent test and hence, a higher ex-ante likelihood of survival.

Bank Specific Factors: Systemic Importance

Observation 2 *N^s is increasing in χ.*

In an interconnected financial system, some institutions by virtue of their credit relationships or their role as conduits for financial capital may impose a higher social cost on the system upon failure. In the model, we capture this characteristic of financial institutions through the externality parameter χ in the social welfare function. It directly follows from Proposition 3 that socially optimal N^s should be higher for systemically more important banks (higher χ). This is so because the expected marginal benefit from running more frequent tests is higher for systemically important financial institutions or whose failure imposes a higher social cost. In practice, bank holding companies that are systemically more important are categorized differently, monitored, and tested more frequently (See Figure (5) in the Appendix).

7 Asynchronous Debt Structure

The crucial feature of our setup is that the agents move sequentially, which enables the regulator to run more tests. If the agents were moving simultaneously, then more than one test will not disclose any new information, making the notion of frequent tests irrelevant. A natural application of our setup is a a maturity mismatch problem where a bank or financial institution has asynchronous debt structure. However, unlike in our benchmark setup, in practice, the creditors do not move continuously. Instead, some fraction of debts mature at a time.

Below, we extend our benchmark setup and add a first stage where the bank chooses the asynchronicity of debt maturity structure M: a fraction $1/M$ of debts mature at dates $0, \frac{1}{M}, \frac{2}{M}, \ldots, \frac{M-1}{M}$. The game is as follows. The bank chooses M. The regulator sees M, and chooses a robustly persuasive stress test $\Gamma(M)$. A creditor whose debt matures date date m, sees all the tests results until then (inclusive). She also receives her private signal. When her debt matures, based on the private signal and the test results, she decide whether to withdraw (attack) or rollover. We do not model the bank's choice of debt duration. The duration of different debt contracts can be identical or different. What is critical here is that all outstanding debts only mature once during the period

[15] Please refer to the ESMA guidelines on stress testing: https://www.esma.europa.eu/sites/default/files/library/esma34-39-897_guidelines_on_liquidity_stress_testing_in_ucits_and_aifs_en.pdf

of liquidity distress ($t \in [0,1]$), and the newly issued debt (which is held by rollover creditors) will all have maturity dates after the end of liquidity distress ($t = 1$).[16]

As before, we assume that the creditors play the worst equilibrium. Abusing notation, we use $\mathcal{P}_f(M, \Gamma)$ to capture the probability of failure in the worst equilibrium given the debt structure M and stress test policy Γ. Accordingly, the regulator's payoff is

$$\Lambda(M, \Gamma) = B - \pi(B + \chi) \cdot \mathcal{P}_f(M, \Gamma) - C(N).$$

We assume that the bank bears the cost of tests. Similar to the regulator the bank also anticipates that the agents will play the worst equilibrium. However, unlike the regulator, the bank ignores the externality it creates on the system when it fails. The bank's payoff is

$$\Lambda_b(M, \Gamma) = B - \pi B \cdot \mathcal{P}_f(M, \Gamma) - C(N)$$

We further assume that if the bank is indifferent, it chooses a lower M.[17]

7.1 Dynamic Liability Structure: A Constraint

If the bank's debt structure is such that no debts mature between two tests, increasing the frequency of tests will not reduce the likelihood of bank failure. Thus, while designing the optimal stress test, the regulator needs to consider the constraint imposed by the asynchronous debt liability structure M. When $N = M$, the analysis is exactly the same as before. That is, if the regulator chooses the stress test policy $(N, k^*(N)^+)$ and the bank chooses $M = N$, then, as before, the only equilibrium in the obedient equilibrium. We use the notation $\mathcal{P}_f(N)$, $\Lambda_b(N)$ and $\Lambda(N)$ when the argument of these expressions are $M = N$ and $\Gamma = (N, k^*(N)^+))$. It is hard to analytically solve for $\mathcal{P}_f(M, \Gamma)$ when $M \neq N$. However, it is easy to see that a regulator will never choose $N > M$ (since higher N costs more and the $(N - M)$ tests are worthless). We show in Lemma (A.2), that for $N \leq M$,

$$\mathcal{P}_f(M, \Gamma) \geq \mathcal{P}_f(N).$$

It follows from this inequality that in equilibrium if the bank chooses $M \leq N^s$, the regulator will choose $\Gamma(M) = (M, k^*(M)^+)$. While, we do not analytically solve for $\Gamma(M)$ when $M > N^s$, we show that on path, the bank will never choose $M > N^s$. Therefore, the bank optimally chooses

[16]In this regard, our model does not consider the demandable deposits. Under deposit insurance coverage, depositor run is unlikely to be the primary reason for bank failure. Moreover, the setup can be generalized to any financial institution engaging in maturity transformation, such as dealer banks and alternative investment funds.

[17]This can be rationalized by some additional cost of issuing more asynchronous debts. Formally, consider lexicographic preference where such costs are of second order importance to the bank.

$M \in [1, N^s]$ to maximize $\Lambda_b(M, (M, k^*(M)^+))$. Define

$$N^e = \min\left\{N \in \mathbb{Z}^+ : k^*(N) - k^*(N+1) \leq \frac{\Delta C(N)(\overline{\theta} - \underline{\theta})}{\pi B}\right\}. \tag{9}$$

Proposition 4 *On the equilibrium path, the bank chooses the debt structure $M = N^e \in [1, N^s]$, the regulator chooses the stress test policy $\Gamma(N^e) = (N^e, k^*(N^e)^+)$, and accordingly, if the shock arrives, the worst equilibrium is the obedient equilibrium.*

The choice of stress test is sub-optimal for social welfare. This sub-optimally arises because the bank does not internalize the externality it imposes on the financial system, and accordingly, it chooses a lower asynchronous debt structure than the social optimal level, which imposes an upper bound on the number of effective stress tests the regulator can run.

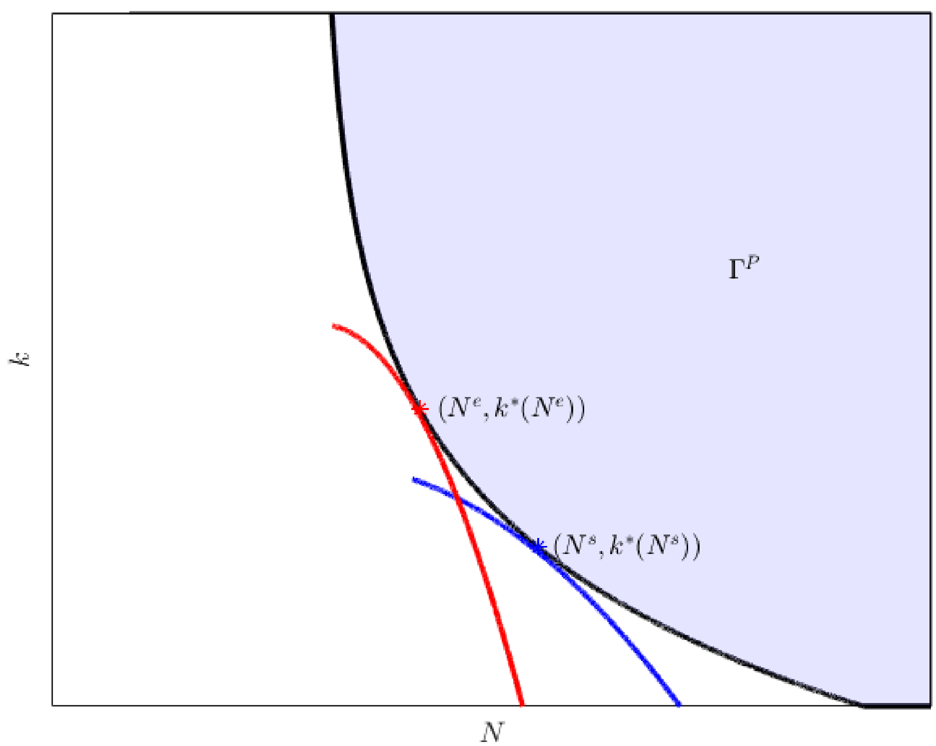


Figure 4: Socially optimal $(N^s, k^*(N^s))$ subject to the persuasive constraint

7.2 Stress Tests and Liquidity Regulation

If we ignore the cost of running more tests, then the first best outcome is when the agents do not attack a bank whenever $\theta \geq 0$. A stress test policy will achieve the first best if the regulator can run at least N_0 tests (since $k^*(N_0) = 0$). However, when the marginal cost of running so many tests is significant, the regulator may choose $N^s < N_0$, or even if the regulator may want $N^s = N_0$, the bank may choose $M < N_0$, and thus, forcing the regulator to run only M tests. This could be the case when the externality χ is high. The regulator may resort to other less costly liquidity regulations which may complement the stress test policy and improve social welfare. In this section we provide such as example, called Liquidity Coverage Ratio (LCR).

Suppose that the bank has debt d of which d_s is debt that matures in the next 30-days. As before, we normalize the possible outflow during a stress event to 1. We assume $d_s < 1 < d$. On the assets side, the bank has the choice to invest in a highly liquid asset (low-yield), a pledgeable asset (medium-yield) and a long-term illiquid asset (high-yield). Suppose the bank has $(1+\eta)$ with $\eta > 0$ amount of assets to be invested in the pledgeable and highly liquid assets. According to LCR policy, the bank is forced to keep λd_s in highly liquid assets. Accordingly, the bank's pledgeable asset is $(1 + \eta - \lambda d_s)$. The investment in liquid assets is associated with a risky component $\tilde{\mu} \sim \mathcal{U}[\underline{\mu}, 0]$ that captures the severity of the shock, and it directly reduces the liquidity of the bank. Moreover, a more severe shock reduces the pledgeablity $\left(1 - \frac{\tilde{\mu}}{\underline{\mu}}\right)$ of the pledgeable assets. Notice that if the shock is most severe ($\underline{\mu}$) pledgeablity becomes zero. Therefore, the liquidity position of the bank is,

$$\theta = \underbrace{\lambda \cdot d_s}_{\text{Liquid asset}} + \underbrace{\tilde{\mu}}_{\text{direct shock}} + \underbrace{\left(1 - \frac{\tilde{\mu}}{\underline{\mu}}\right)}_{\text{pledgeablity}} \cdot \underbrace{(1 + \eta - \lambda \cdot d_s)}_{\text{pledgeable asset}}.$$

This implies $\theta \sim \mathcal{U}[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta} = \lambda \cdot d_s + \underline{\mu}$, $\overline{\theta} = (1 + \eta)$. Notice that a higher λ increases $\underline{\theta}$. This means if the regulator increases the LCR, the prior belief about the bank's fundamental improves. Below, we show how this affects welfare.[18]

Proposition 5 *The social welfare $\Lambda(N^s, (N^s, k^*(N^s))$ (without debt structure constraint) and $\Lambda(N^e, (N^e, k^*(N^e))$ (with debt structure constraint) increases with LCR (λ).*

The formal proof is relegated to the appendix. Below, we provide the main argument. A higher λ increases $\underline{\theta}$, which reduces the probability of failure of the bank (See (4)), and thus, directly improves welfare. Moreover, this increases the marginal benefit of increasing the frequency of tests (if there is no debt structure constraint), and thus, indirectly increases the equilibrium choice N^s. A higher N^s further reduces the probability of failure (by lowering $k^*(N)$), but it also increases the cost. It follows from envelope condition that this indirect effect on welfare is non-negative. Therefore, the direct and indirect effect together improves welfare. When the bank imposes an upper bound on number of tests by choosing $M = N^e$, this envelope condition does not apply. However, since $M = N^e$ is chosen such that it maximizes $\Lambda_b(M, (M, k^*(M))$ (See Proposition 4), the increase

[18] Mandating very high values for regulatory ratios λ could be costly as well. However, in this stylized example, we ignore such costs and only focus on how LCR can complement stress tests.

in cost is lower than the social benefit from a higher N^e (because Λ_b ignores the externality χ). Therefore, this results in a net increase in social welfare. This means a higher LCR encourages the bank to choose a higher asynchronous debt structure, and thus, relaxes the constraint on the regulator, and indirectly improves welfare. Notice that both the direct and indirect effects improves welfare. This shows that regulatory policies such as LCR promote financial stability by changing the prior of belief about the fundamental through an increase of $\underline{\theta}$.

8 Conclusion

Policy discussions on financial regulation often argue for more frequent stress tests, especially for systemically important banks, and during times of higher likelihood of adverse shocks. The theoretical literature has established how a sufficiently strong stress test robustly dissuades the agents from running on a bank. However, the frequency of testing as a policy lever has not been explored theoretically. This paper establishes how higher frequency helps, characterizes the optimal frequency, and discusses what determines the optimal frequency.

During a bank run, the agents may not move simultaneously (for instance, when the debts mature at different dates). This asynchronous move enables the regulator to conduct multiple tests and disclose new information based on past actions. We show that when the regulator conducts more tests (higher N), she can weaken the strengths while keeping the tests robustly persuasive (lower $k^*(N)$). The bank is less likely to pass when the test is stronger. Therefore, by reducing the required strength of each test, more frequent tests increase the probability that a bank can survive the tests. However, it could be costly to run more tests. We also show that $k^*(N)$ is convex, thereby ensuring a unique optimal frequency.

We discuss some policy implications of this theory. For instance, one may think that the regulator should increase the strength of stress tests under worse market conditions. However, we show that it is possible that under worse market conditions, the optimal stress test policy involves more frequent but more lenient tests. We also show an example of how other regulatory policies may complement the stress test policy.

Appendix: Omitted Proofs

Proof of Lemma 1.

Under monotone strategies, $\rho_i(s_i, \zeta^n)$ decreases if (1) s_i increases, or (2) $\zeta_{n'} = 1$ rather than $\zeta_{n'} = 0$ for $n' \leq n$. Consider $\theta'' > \theta'$. If the agents play monotone strategies, then for any given ζ^n, there is less attack under θ'' than under θ'. Therefore, the bank is more likely to pass the next test. If the bank passes (or fails) the next test under both θ'' and θ', then there will be less attack under θ'' than under θ' from the agents who move after the next test and before the next to next test (follows from the first property of monotone strategy). On the other hand, if the bank passes the next test under θ'' but fails under θ', then also there will be less attack under θ'' (follows from the second property of monotone strategy). Continuing this argument, we can see that if the bank survives under θ', then it will also survive under $\theta'' > \theta'$. Thus, in a PBE where the agents play monotone strategies, there exists a threshold θ^* such that the bank survives iff $\theta \geq \theta^*$.

Given log-concave signals, this means an agent with higher signal believes that the regime is more likely to survive. Therefore, given any ζ^n, if an agent with signal s_i' prefers not attacking, so will an agent with signal $s_i'' > s_i'$. This implies that, in equilibrium, the agents will play symmetric cutoff strategies $s_n^*(\zeta^n)$ for any ζ^n. Let $\tilde{\mathbf{s}}^* = \{s_n^*(\zeta^n)\}_{n=1}^N$ be the equilibrium cutoffs, and $\tilde{\mathcal{E}}(\Gamma)$ be the set of equilibrium $\tilde{\mathbf{s}}^*$ given a stress test policy Γ.

Notice that for any $\tilde{\mathbf{s}}^*$, there exists θ^* such that the bank survive in the end iff $\theta \geq \theta^*$. $\hat{\mathbf{s}} \in \tilde{\mathcal{E}}(\Gamma)$ is the worst equilibrium under stress test policy Γ iff $\theta^*(\hat{\mathbf{s}}, \Gamma) \geq \theta^*(\tilde{\mathbf{s}}^*, \Gamma)$ for all $\tilde{\mathbf{s}}^* \in \tilde{\mathcal{E}}(\Gamma)$. To see why $\hat{\mathbf{s}}$ must have the two properties, consider any equilibrium $\tilde{\mathbf{s}}^* = \{s_n^*(\zeta^n)\}$. Consider an alternative cutoff strategy that satisfies the first property. This means under this alternative strategy, the agents behave optimally once the bank fails a test. Moreover, for ζ^n such that $\zeta_{n'} = 1$ for all $n' \leq n$, the marginal agent $s_n^*(\zeta^n)$ is less optimistic about the survival of the regime (since there will be more attacks if the bank fails any subsequent test). Therefore, we can construct a cutoff strategy with higher thresholds, which results in a higher θ^*.

Since, in the worst monotone PBE, agents always attack whenever a bank fails a test (property 1), we simplify the notation in the main text and describe the equilibrium by N thresholds $\mathbf{s}^* = \{s_n^*\}_{n=1}^N$ where s_n^* is the threshold for groups that move after the n-th test and before the $(n+1)$-th test, given that the bank has passed all first n stress tests. ■

Proof of Proposition 1.

For a given stress test policy $\Gamma = (n, k)$, we characterize the equilibrium $\mathbf{s}^* = \{s_n^*\}_{n=1}^n$ using four simple steps.

Step 1 (Passing the tests and survival): Given $\mathbf{s}^*$, there exists a weakly increasing sequence

$$k = \underline{\theta}_0 \leq \underline{\theta}_1 \leq \ldots \leq \underline{\theta}_{n-1} \leq \underline{\theta}_N = \theta^*.$$

The bank passes the first n tests iff $\theta \geq \underline{\theta}_{n-1}$, and survives in the end iff $\theta \geq \theta^*$. To see how this thresholds are determined, notice that for any θ, the size of attack from group n' is $\frac{1}{N} F\left(\frac{s^*_{n'}-\theta}{\sigma}\right)$. The bank will pass the n-th test ($\zeta_n = 1$) iff

$$\theta - \frac{1}{N}\sum_{n'=1}^{n-1} F\left(\frac{s^*_{n'} - \theta}{\sigma}\right) \geq k\left(1 - \frac{n-1}{N}\right).$$

Since the LHS is increasing in θ, there exists $\tilde{\theta}_{n-1}$ such the above holds true iff $\theta \geq \tilde{\theta}_{n-1}$. Therefore, the bank passes all the first n stress tests iff

$$\theta \geq \underline{\theta}_{n-1} := \max\{\tilde{\theta}_0, \tilde{\theta}_1, \ldots \tilde{\theta}_{n-1}\}.$$

Finally, we define $\tilde{\theta}_N$ such that when $\theta = \tilde{\theta}_N$, the aggregate attack from all the groups is same as θ

$$\tilde{\theta}_N - \frac{1}{N}\sum_{n'=1}^{N} F\left(\frac{s^*_{n'} - \tilde{\theta}_N}{\sigma}\right) = 0.$$

Thus, the regime passes all the n tests and survive in the end iff $\theta \geq \theta^*$ where

$$\theta^* = \underline{\theta}_N = \max\{\tilde{\theta}_0, \tilde{\theta}_1, \ldots, \tilde{\theta}_N\}.$$

Step 2 (Indifference): After the bank passes the first n stress tests, group n learns that $\theta \geq \underline{\theta}_{n-1}$. In equilibrium, a agent i in group n wants to attack iff $s_i < s^*_n$. Notice that he believes that the bank will survive with probability

$$P(\theta \geq \theta^* | s_i, \theta \geq \underline{\theta}_{n-1}) = \frac{F\left(\frac{s_i - \theta^*}{\sigma}\right)}{F\left(\frac{s_i - \underline{\theta}_{n-1}}{\sigma}\right)}.$$

It follows from log-concavity that this belief is increasing in s_i. The lowest signal a agent i in group n may receive is $s_i = \underline{\theta}_{n-1} - \sigma/2$. If $s^*_n > \underline{\theta}_{n-1} - \sigma/2$ then the marginal agent must believe that the probability of survival

$$\frac{F\left(\frac{s^*_n - \theta^*}{\sigma}\right)}{F\left(\frac{s^*_n - \underline{\theta}_{n-1}}{\sigma}\right)} = p.$$

This makes the marginal agent indifferent between attacking and not attacking. While all the agents with $s_i > s^*_n$ prefer not attacking and all the agents with $s_i < s^*_n$ prefer attacking. If $s^*_n = \underline{\theta}_{n-1} - \sigma/2$, then no agent in group n attacks. We say that group n is persuaded. In this case, the marginal agent may believe the probability of survival is weakly higher than p.

Step 3 (First persuaded group): Suppose that group n is persuaded $s^*_n = \underline{\theta}_{n-1} - \sigma/2$. Then, s^*_n believes θ cannot be higher than $\underline{\theta}_{n-1}$. Therefore, if $\theta^* > \underline{\theta}_{n-1}$, then s^*_m will always attack. Thus, group n is persuaded iff $\theta^* = \underline{\theta}_{n-1}$. Since $\underline{\theta}_n$ is weakly increasing, this means

$\underline{\theta}_{n-1} = \underline{\theta}_n = \ldots = \underline{\theta}_{N-1} = \underline{\theta}_N = \theta^*$. Accordingly, $s^*_{n'} = \underline{\theta} - \sigma/2$ for all $n' \geq n$. Thus, if in equilibrium group n is persuaded then it must be that all the subsequent groups are persuaded. Thus, for any $\mathbf{s}^*$, there is $\overline{n} \in \{1, 2 \ldots N+1\}$ that captures the first group that is persuaded. If no group is persuaded, we use the convention that $\overline{n} = N+1$.

Step 4 (Non-persuaded groups): Consider $n < \overline{n}$. Notice the indifference condition of the marginal agent from a non-persuaded group. Since $\underline{\theta}_{n-1}$ is weakly increasing, it follows from log-concavity that s^*_n is weakly decreasing in n. That is, fewer agents attack from later groups. This implies $\tilde{\theta}_n$ is strictly increasing in n. To see this, notice that $\tilde{\theta}_n \leq \tilde{\theta}_{n-1}$ iff

$$F\left(\frac{s^*_n - \tilde{\theta}_{n-1}}{\sigma}\right) \leq k.$$

If this holds true, then all subsequent groups will have even less attack. Therefore, if the bank passes the nth test it will pass the subsequent tests and survive in the end, which implies $\underline{\theta}_{n-1} = \theta^*$ contradicting the fact that group n is not persuaded. Therefore, for all non-persuaded groups $n < \overline{n}$, we have $\underline{\theta}_{n-1} = \tilde{\theta}_{n-1}$. ■

Proof of Theorem 2.

Define

$$y(N,k) \equiv \min_{x \in [k,1]} G(x,N,k). \tag{A.1}$$

Since $G(x,N,k)$ is increasing in N, then, by definition, $y(N,k)$ is increasing in N. To see this, consider $N' > N$, and define,

$$\underline{x}(N,k) \equiv \arg\min_{x \in [k,1]} G(x,N,k). \tag{A.2}$$

Then, we have

$$y(N,k) = G(\underline{x}(N,k), N, k) \leq G(\underline{x}(N',k), N, k) < G(\underline{x}(N',k), N', k) = y(N',k)$$

The first inequality follows from the definition of $\underline{x}(N,k)$ and the second from the monotonicity of G with respect to N. A similar argument can be used to show that $y(N,k)$ is increasing in k.

Consider $y(N, k=0)$ and note that $\lim_{N\to\infty} y(N,0) = 1 > p$. Since $y(N,0)$ is increasing in N, there exists a unique $N_0 \geq 1$ such that $y(n, k=0) > p$ if and only if $n \geq N_0$.[19] Alternatively,

$$N_0 = \min\{N \in \mathbb{Z}^+ | y(N, k=0) \geq p\}$$

[19] For the details of the existence of N_0, see Basak and Zhou (2020). For example, if σ is sufficiently large, then $N_0 = 1$.

Therefore, for any $N \leq N_0 - 1$, $k^*(N) > 0$. Let N' be any number in the range of $(N, N_0]$. We have

$$y(N, k^*(N)) = p = y(N', k^*(N')) < y(N', k^*(N)).$$

Since $y(N, k)$ is increasing in k, it must be that $k^*(N') < k^*(N)$. Therefore, for $N \in [1, N_0 - 1]$, $k^*(n)$ is strictly decreasing in N; and, for all $N \geq N_0$, $k^*(N) = 0$. This completes the first two parts of the Theorem. Next, we prove the discrete convexity of $k^*(N)$ for $N \in [1, N_0 - 1]$.

Lemma A.1 *$y(N, k)$ is quasiconcave in (N, k).*

Proof of Lemma A.1. We first prove that $G(x, N, k)$ is quasiconcave in (N, k). Consider (N, k), (N', k'), and any $\lambda \in [0, 1]$ and fix x. Define

$$k'' = \lambda k + (1 - \lambda)k' \quad \text{and} \quad N'' = \lambda N + (1 - \lambda)N'.$$

Without loss of generality, let $G(x, N, k) \geq G(x, N', k')$. Note that, $G(x, N, k) \geq G(x, N', k')$ implies $\frac{x-k}{N\sigma} \leq \frac{x-k'}{N'\sigma}$, which, in turn, implies

$$x(N' - N) \leq kN' - k'N.$$

Then, we have

$$\begin{aligned}\frac{x - k''}{N''\sigma} - \frac{x - k'}{N'\sigma} &= \frac{1}{N''N'\sigma}\left(x(N' - N'') - (k''N' - k'N'')\right) \\ &= \frac{\lambda}{N''N'\sigma}\left(x(N' - N) - (kN' - k'N)\right) \leq 0\end{aligned}$$

Therefore, $G(x, N, k)$ is quasiconcave in (N, k) since

$$G(x, N'', k'') \geq G(x, N', k') = \min\{G(x, N, k), G(x, N', k')\}.$$

Next, we prove the quasiconcavity of $y(N, k)$ based on the fact the $G(x, N, k)$ is quasiconcave in (N, k). Note that, since $G(x, N, k) \leq 1$ for any x, N and k,

$$y(N, k) = \min_{x \in [k,1]} G(x, N, k) = \min_{x \in [0,1]} \{G(x, N, k) + I_k(x)\}$$

where $I_k(x) = \mathbb{1}(x < k)$. Therefore, it is sufficient to show $G(x, N, k) + I_k(x)$ is quasiconcave in (N, k) because the infimum of a quasiconcave function is quasiconcave.[20] Without loss of generality let us consider, $k > k'$. Given that, only two cases are possible.

Case 1. Either $x < k' < k$ or $k' < k \leq x$. In either case,

$$I_{k''}(x) = I_k(x) = I_{k'}(x) = \text{constant}$$

[20]see Boyd, Boyd and Vandenberghe (2004).

Therefore, from the quasiconcavity of $G(x, N, k)$, we have,

$$\begin{aligned} G(x, N'', k'') + I_{k''}(x) &\geq \min\{G(x, N, k) + I_{k''}(x), G(x, N', k') + I_{k''}(x)\} \\ &= \min\{G(x, N, k) + I_k(x), G(x, N', k') + I_{k'}(x)\} \end{aligned}$$

Case 2. $k' \leq x < k$. Now we have, $I_{k'}(x) = 0$ and $I_k(x) = 1$. Given that $G(x, N', k') \in [0, 1]$ for all $x \in [k', 1]$,

$$\min\{G(x, N, k) + I_k(x), G(x, N', k') + I_{k'}(x)\} = G(x, N', k').$$

Clearly, if $\lambda \in [0, 1]$ is such that $k'' > x \geq k'$, then $I_{k''}(x) = 1$ and

$$G(x, N'', k'') + I_{k''}(x) \geq G(x, N', k') = \min\{G(x, N, k) + I_k(x), G(x, N', k') + I_{k'}(x)\}$$

However, if $x \geq k'' \geq k'$, then,

$$\frac{x - k''}{\sigma N''} - \frac{x - k'}{\sigma N'} = \frac{1}{NN''\sigma} \cdot \left(x(N' - N'') - (k''N' - k'N'')\right).$$

Substituting ($k'' = \lambda k + (1 - \lambda)k'$ and $N'' = \lambda N + (1 - \lambda)N'$, we have

$$\frac{x - k''}{\sigma N''} - \frac{x - k'}{\sigma N'} = \frac{\lambda}{NN''\sigma} \cdot \left(x(N' - N) - (kN' - k'N)\right).$$

If $N' - N > 0$, using $x < k$, we have

$$\frac{x - k''}{\sigma N''} - \frac{x - k'}{\sigma N'} < \frac{\lambda}{NN''\sigma} \left(k(N' - N) - (kN' - k'N)\right) = \frac{\lambda}{N''\sigma} \cdot (k' - k) < 0.$$

Otherwise, if $N' - N \leq 0$, using $x \geq k'$, we have

$$\frac{x - k''}{\sigma N''} - \frac{x - k'}{\sigma N'} \leq \frac{\lambda}{NN''\sigma} \cdot \left(k'(N' - N) - (kN' - k'N)\right) = \frac{\lambda N'}{N''N\sigma} \cdot (k' - k) < 0.$$

Therefore, $G(x, N'', k'') > G(x, N', k')$, which implies

$$G(x, N'', k'') + I_{k''}(x) \geq G(x, N', k') = \min\{G(x, N, k) + I_k(x), G(x, N', k') + I_{k'}(x)\},$$

completing the proof that $y(N, k) = \min_{x \in [0,1]}\{G(x, N, k) + I_k(x)\}$ is quasiconcave in (N, k). ■

By definition, $\mathbf{\Gamma}^P = \{(N, k) : y(N, k) \geq p\}$. Consider $(N, k^*(N)), (N', k^*(N') \in \mathbf{\Gamma}^P$. By Lemma A.1, $y(N, k)$ is quasiconcave, which implies that, for $\lambda \in [0, 1]$,

$$y(\lambda N + (1 - \lambda)N', \lambda k^*(N) + (1 - \lambda)k^*(N')) \geq p. \tag{A.3}$$

That means, $(\lambda N + (1 - \lambda)N', \lambda k^*(N) + (1 - \lambda)k^*(N')) \in \mathbf{\Gamma}^P$, proving that the set $\mathbf{\Gamma}^P$ is convex. By the definition of $k^*(N)$ (see (5)), $k^*(N) = \inf\{k : y(N, k) \geq p\}$. Given that, (A.3) implies

$$k^*(\lambda N + (1 - \lambda)N') \leq \lambda k^*(N) + (1 - \lambda)k^*(N'),$$

thereby proving the convexity of $k^*(N)$. As k^* is defined on $\mathbb{Z}^+$, it is discrete convex as we have shown that the extension of k^* that defines on $\mathbb{R}^+$ is convex (see Boyd, Boyd and Vandenberghe (2004)). By the definition of discrete convexity, $k^*(N) - k^*(N+1)$ is decreasing in N. ■

Proof of Proposition 2. Define the function $\tilde{G}$ as follows,[21]

$$\tilde{G}(x,\sigma,n,k) = G(x,\sigma,n,k) + I_k(x)$$

where $I_k(x) = \mathbb{1}(0 \leq x < k)$ is an indicator function that is equal to 1 for all $x \in [0,k)$ and 0 otherwise.

Define the minimizer of $\tilde{G}$,

$$\underline{x}(\sigma,N,k) = \arg\min_{x\in[0,1]}\{\tilde{G}(x,\sigma,N,k)\} = \arg\min_{x\in[k,1]}\{G(x,\sigma,N,k)\}$$

Consider $\sigma_1 > \sigma_0$. Note that $N_0(\sigma_1) < N_0(\sigma_0)$ since $G(x,\sigma,N,k)$ is increasing in σ and N. Hence, there are several cases to consider, (1) $N < N_0(\sigma_1)$ (2) $N \in [N_0(\sigma_1), N_0(\sigma_1)]$ and (3) $N \geq N_0(\sigma_0)$. Case (2) and (3) are trivial because we have

$$k^*(N,\sigma_0) \geq k^*(N,\sigma_1) = 0$$

.

We now focus on Case(1).

Case 1: $N < N_0(\sigma_1)$

From the definition of $\underline{x}(\sigma,N,k) \in [k,1]$,

$$\tilde{G}(\underline{x}(\sigma_0,N,k^*(N,\sigma_0)),\sigma_0,N,k^*(N,\sigma_0)) = \tilde{G}(\underline{x}(\sigma_1,N,k^*(N,\sigma_1)),\sigma_1,N,k^*(N,\sigma_1)) = p$$

Note that,

$$\tilde{G}(\underline{x}(\sigma_1,N,k^*(N,\sigma_1)),\sigma_1,N,k^*(N,\sigma_1)) = p > \tilde{G}(\underline{x}(\sigma_1,N,k^*(N,\sigma_1)),\sigma_0,N,k^*(N,\sigma_1)) \quad \text{(A.4)}$$

The strict inequality follows from $\tilde{G}(x,\sigma,N,k)$ is increasing in σ for all $x \in [k,1]$

Again, using the definition of $\underline{x}(\sigma,N,k)$

$$\tilde{G}(\underline{x}(\sigma_1,N,k^*(N,\sigma_1)),\sigma_0,N,k^*(N,\sigma_0)) \geq \tilde{G}(\underline{x}(\sigma_0,N,k^*(N,\sigma_0)),\sigma_0,N,k^*(N,\sigma_0)) = p \quad \text{(A.5)}$$

[21] Since, $\lim_{x\to k} G(x,\sigma,N,k) = 1$, $\tilde{G}$ is continuous in x for $x \in [0,1]$.

Combining Conditions (A.4) and (A.5)

$$\tilde{G}(\underline{x}(\sigma_1, N, k^*(N, \sigma_1)), \sigma_0, N, k^*(N, \sigma_0)) > \tilde{G}(\underline{x}(\sigma_1, N, k^*(N, \sigma_1)), \sigma_0, N, k^*(N, \sigma_1))$$

But $\tilde{G}(x, \sigma_0, N, k)$ is weakly increasing in k for all $x \in [0, 1]$. Therefore, it must be that for $\sigma_1 > \sigma_0$, the above implies

$$k^*(N, \sigma_0) > k^*(N, \sigma_1)$$

Following an exactly similar argument we can prove that for $p_1 > p_0$ and $N \in [1, N_0(p_0) - 1]$,

$$k^*(N, p_1) > k^*(N, p_0)$$

for $N \geq [N_0(p_0), N_0(p_1) - 1]$, the above is trivially true as

$$k^*(N, p_1) > k^*(N, p_0) = 0$$

and for $N \geq N_0(p_1)$,

$$k^*(N, p_1) = k^*(N, p_0) = 0$$

■

Proof of Proposition 3.

Notice that $\Lambda(N, k)$ is decreasing and separable in N and k and it is quasi-concave. Therefore, given the convex persuasive constraint, there always exists a social optimal. Given any N, the regulator can always improve Λ by reducing k. However, if $k \leq k^*(N)$, then the stress tests are no longer persuasive. Therefore, for any N, the social optimal stress test must have $k = k^*(N)^+$. Notice that

$$\mathcal{P}_f(N, k^*(N)^+) = Pr(\theta < k^*(N)^+) = \frac{k^*(N) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}.$$

Substituting this in the regulator's objective function, we get

$$\Lambda(N, k^*(N)^+) = B - \frac{\pi(B + \chi)k^*(N)}{\overline{\theta} - \underline{\theta}} - C(N),$$

which is concave in N since both $k^*(N)$ (See Theorem 1) and $C(N)$ are convex. Therefore, the maximizer N^s must be such that,

$$\Lambda(N^s, k^*(N^s)) \geq \max\{\Lambda(N^s + 1, k^*(N^s + 1)), \Lambda(N^s - 1, k^*(N^s - 1))\}$$

Defining the marginal cost as $\Delta C(N) = C(N + 1) - C(N)$, this implies,

$$\frac{(k^*(N^s) - k^*(N^s + 1))\pi(B + \chi)}{\overline{\theta} - \underline{\theta}} \leq \Delta C(N^s) \leq \frac{(k^*(N^s - 1) - k^*(N^s))\pi(B + \chi)}{\overline{\theta} - \underline{\theta}}$$

Define $H : \mathbb{Z}^+ \to \mathbb{R}$

$$N \mapsto k^*(N) - k^*(N + 1)$$

The conditions simplify to,

$$H(N^s) \leq \frac{\Delta C(N^s)(\overline{\theta} - \underline{\theta})}{\pi(B+\chi)} \leq H(N^s - 1)$$

Since $H(N)$ is weakly decreasing,[22] this can also be written as,

$$N^s = \min\left\{N \in \mathbb{N} : H(N) \leq \frac{\Delta C(N)(\overline{\theta} - \underline{\theta})}{\pi(B+\chi)}\right\} \tag{N^s}$$

■

Proof of Proposition 4. A regulator will never choose $N > M$ (since higher N costs more and the $(N - M)$ tests are worthless). Consider $N \leq M$.

Lemma A.2 *For any robustly persuasive* $\Gamma = (N, k)$*, and debt structure* $M \geq N$*,* $\mathcal{P}_f(M, \Gamma) \geq \mathcal{P}_f(N)$.

Proof.

If $M \geq N_0$ the claim is trivially true since $k^*(M) = 0$ and k cannot be lower than 0. Consider $M < N_0$. For $N = M$, again the claim holds true with equality by definition. Suppose that the regulator chooses $N < M$. Note that if $N < M$, then at least two groups ($n \geq 2$) move between the first test and the second test. Suppose that if the bank passes the second test, all the creditors moving later will be persuaded, that is, $\theta^* = \underline{\theta}_1$. Then, it follows from (ζ_n) and (S) that

$$\theta^* - \frac{n}{M} F\left(\frac{s_1^* - \theta^*}{\sigma}\right) = k\left(1 - \frac{n}{M}\right). \tag{ζ_n}$$

This implies

$$\theta^* - k = \frac{n}{M}\left(F\left(\frac{s_1^* - \theta^*}{\sigma}\right) - k\right).$$

Substituting this in (I_n) we can say that the marginal agent after the first test believes the bank will survive in the end with probability

$$\frac{F\left(\frac{s_1^* - \theta^*}{\sigma}\right)}{F\left(\frac{s_1^* - \theta^*}{\sigma} + \frac{\theta^* - k}{\sigma}\right)} = \frac{x}{F\left(F^{-1}(x) + \frac{n(x-k)}{M\sigma}\right)} = G(x, M/n, k) < \frac{x}{F(F^{-1}(x) + \frac{x-k}{M\sigma})}.$$

The inequality follows from $n \geq 2$. Therefore, for $k \leq k^*(M)$ there exists x^* such that

$$G(x^*, M, k) = p > G(x^*, M/n, k)$$

[22] To see this, consider $H(N+1) - H(N) = 2\left(\frac{k^*(N+1) + k^*(N-1)}{2} - k^*(N)\right) \geq 0$. Since $k^*(N)$ is convex,

$$\alpha k^*(N+1) + (1-\alpha)k^*(N-1) \geq k^*(\alpha(N+1) + (1-\alpha)(N-1))$$

for all $\alpha \in [0, 1]$ and particularly for $\alpha = 0.5$.

We can find $x^{*'} \in (x^*, 1)$ (and s_1^*, θ^*) such that

$$\frac{F\left(\frac{s_1^*-\theta^*}{\sigma}\right)}{F\left(\frac{s_1^*-k}{\sigma}\right)} = G(x^{*'}, M/n, k) = p$$

Hence, even if all the groups moving after the second test are obedient, there exists at least one equilibrium in which the group moving before the second test will adopt a strategy s_1^*. If $N < M < N_0$, then $k^*(M)^+$ is not robustly persuasive. Consequently, $\mathcal{P}_f(M, \Gamma) \geq \mathcal{P}_f(N)$. ■

Next, we find the optimal stress test given any choice of asynchronous debt structure M.

Case 1 ($M \leq N^s$) : For a debt structure M and a robustly persuasive policy $\Gamma = (N, k)$ with $N < M \leq N^s$,

$$\Lambda(M, (N, k)) \leq \Lambda(N) < \Lambda(M).$$

The first inequality follows from the above Lemma and the second inequality follows from the fact that $\Lambda(N)$ is increasing in N for $N \leq N^s$ (since $\Lambda(N)$ is concave and maximized at N^s). This shows that for any debt structure $M \leq N^s$ the bank chooses, the regulator will choose $\Gamma(M) = (M, k^*(M)^+)$.

Case 2 ($M > N^s$) : The regulator may optimally choose $\Gamma(M)$ with $N(M) < M$. Nevertheless, for any robustly persuasive $\Gamma = (N, k)$, it follows from the above lemma that $\Lambda_b(M, \Gamma(M)) \leq \Lambda_b(N(M))$.

- If $N(M) \leq N^s$, then $\Lambda_b(M, \Gamma(M)) \leq \Lambda_b(N(M))$, that is, the bank can simply chooses $M = N(M)$ and get at least as high payoff. Moreover, since the bank prefers a lower M for breaking ties, the bank will choose $N(M)$ over such M.
- On the other hand, if $N(M) > N^s$, then $\Lambda_b(M, \Gamma(M)) \leq \Lambda_b(N(M)) < \Lambda_b(N^s)$ (Since $\Lambda_b(N)$ is decreasing in N for $N \geq N^s$). Therefore, the bank will choose N^s over such M.

This shows that in equilibrium, if the bank chooses $M \leq N^s$, the regulator will choose $\Gamma(M) = (M, k^*(M)^+)$. While, we do not analytically solve for $\Gamma(M)$ when $M > N^s$, we show that on path, the bank will never choose $M > N^s$. Therefore, the bank optimally chooses $M \in [1, N^s]$ to maximize $\Lambda_b(M, (M, k^*(M)^+))$.

Proof of Proposition 5.

Suppose that the regulator is free to choose N (no constraint by asynchronous debt structure).

The optimal social welfare in this case is

$$\Lambda(N^s, k^*(N^s)) = B - \pi(B+\chi)\frac{k^*(N^s) - \underline{\theta}}{(\overline{\theta} - \underline{\theta})} - C(N^s)$$

First, note that $\underline{\theta} = \lambda d_s$. For a given d_s, it is sufficient to show that $\Lambda(.)$ is increasing in $\underline{\theta}$. Consider $\underline{\theta}^1 > \underline{\theta}$ and the corresponding $N_1^s = N^s(\underline{\theta}^1) \geq N^s(\underline{\theta}) = N^s$.

$$\begin{aligned}\Delta\Lambda &= \Lambda(N_1^s, k^*(N_1^s) - \Lambda(N^s, k^*(N^s)) \\ &= \pi(B+\chi)\cdot\left(\frac{k^*(N^s) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} - \frac{k^*(N_1^s) - \underline{\theta}^1}{\overline{\theta} - \underline{\theta}^1}\right) - (C(N_1^s) - C(N^s))\end{aligned}$$

Add and subtract $\pi(B+\chi)\frac{k^*(N^s) - \underline{\theta}^1}{\overline{\theta} - \underline{\theta}^1}$,

$$\begin{aligned}\Delta\Lambda = &\underbrace{\pi(B+\chi)\cdot\left(\frac{k^*(N^s) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} - \frac{k^*(N^s) - \underline{\theta}^1}{\overline{\theta} - \underline{\theta}^1}\right)}_{\text{direct effect}>0} \\ &+ \underbrace{\left(\pi(B+\chi)\cdot\left(\frac{k^*(N^s) - k^*(N_1^s)}{\overline{\theta} - \underline{\theta}^1}\right) - (C(N_1^s) - C(N^s))\right)}_{\text{indirect effect}}\end{aligned}$$

We show that the indirect effect is non-negative by envelope condition. To see this, notice from the definition of N^s in Proposition (3), N^s is such that,

$$\pi(B+\chi)\frac{(k^*(N^s) - k^*(N^s+1))}{(\overline{\theta} - \underline{\theta})} \leq C(N^s+1) - C(N^s).$$

Since $k^*(N)$ and $C(N)$ are convex, for any $N \leq N'$

$$(\pi B+\chi)\frac{(k^*(N) - k^*(N'))}{(\overline{\theta} - \underline{\theta})} \geq C(N') - C(N)$$

Therefore, since $N_1^s \geq N^s$, the indirect effect

$$\left(\pi(B+\chi)\cdot\left(\frac{k^*(N^s) - k^*(N_1^s)}{\overline{\theta} - \underline{\theta}^1}\right) - (C(N_1^s) - C(N^s))\right) \geq 0$$

Therefore, $\Delta\Lambda > 0$.

Next, consider the case where the bank chooses $M = N^e$ and forces the regulator to choose $N = M$ (as in proposition 4). Let $N_1^e = N^e(\underline{\theta}^1) \geq N^e(\underline{\theta}) = N^e$. As before, we can write

$$\begin{aligned}\Delta\Lambda = &\underbrace{\pi(B+\chi)\cdot\left(\frac{k^*(N^e) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} - \frac{k^*(N^e) - \underline{\theta}^1}{\overline{\theta} - \underline{\theta}^1}\right)}_{\text{direct effect}>0} \\ &+ \underbrace{\left(\pi(B+\chi)\cdot\left(\frac{k^*(N^e) - k^*(N_1^e)}{\overline{\theta} - \underline{\theta}^1}\right) - (C(N_1^e) - C(N^e))\right)}_{\text{indirect effect}}\end{aligned}$$

We can further breakdown the indirect effect as

$$\pi B \cdot \left(\frac{k^*(N^e) - k^*(N_1^e)}{\overline{\theta} - \underline{\theta}^1} \right) - (C(N_1^e) - C(N^e)) + \pi\chi \cdot \left(\frac{k^*(N^e) - k^*(N_1^e)}{\overline{\theta} - \underline{\theta}^1} \right)$$

From definition of N^e in Proposition 4, N^e is such that

$$\pi B \frac{(k^*(N^e) - k^*(N^e + 1))}{(\overline{\theta} - \underline{\theta})} \leq C(N^e + 1) - C(N^e)$$

Since $k^*(N)$ and $C(N)$ are convex, for any $N \leq N'$

$$\pi B \frac{(k^*(N) - k^*(N'))}{(\overline{\theta} - \underline{\theta})} \geq C(N') - C(N)$$

Therefore, since $N^e \leq N_1^e$,

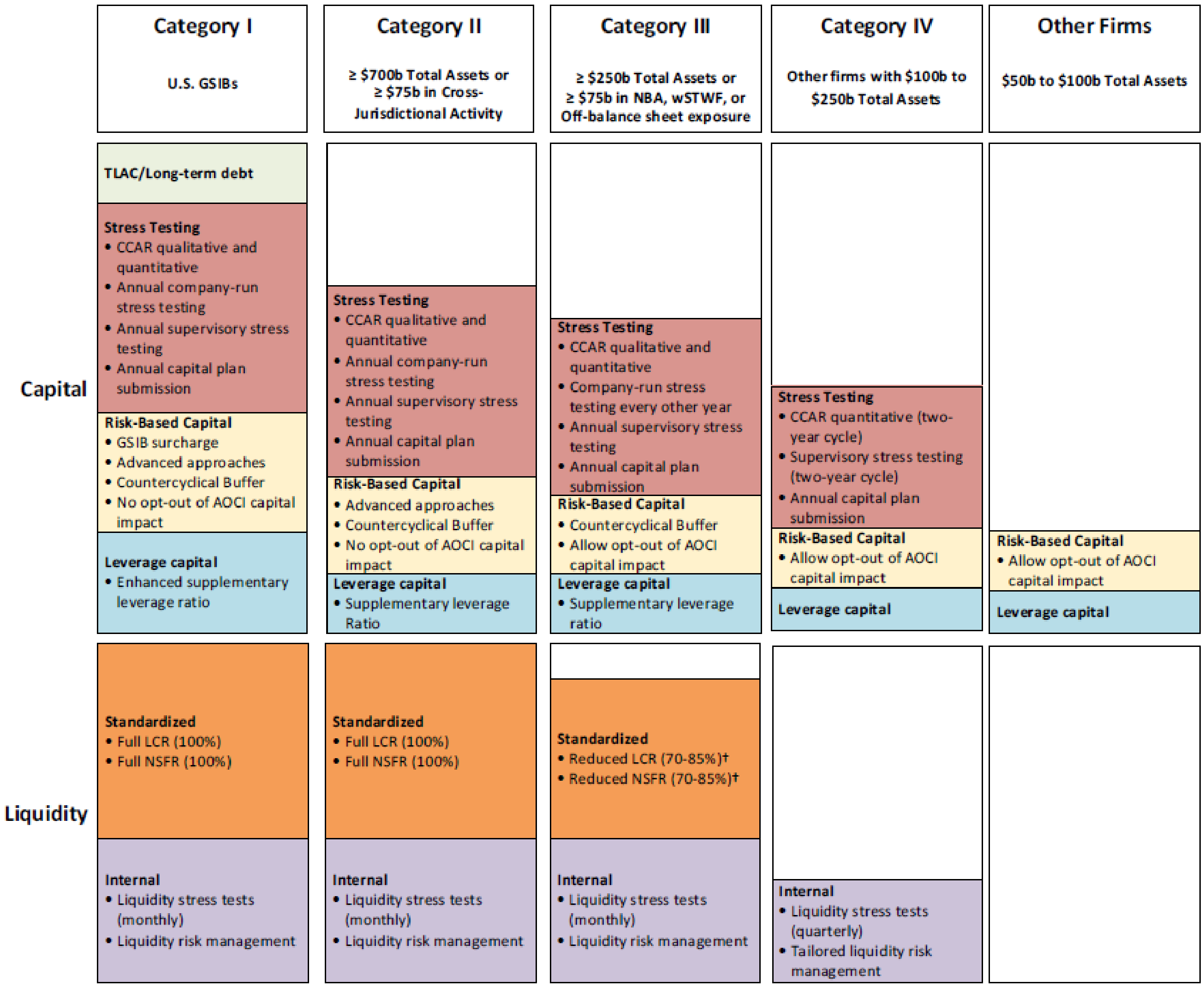
$$\pi B \cdot \left(\frac{k^*(N^e) - k^*(N_1^e)}{\overline{\theta} - \underline{\theta}^1} \right) - (C(N_1^e) - C(N^e))$$

Moreover, since $k^*(N)$ is decreasing,

$$\pi\chi \cdot \left(\frac{k^*(N^e) - k^*(N_1^e)}{\overline{\theta} - \underline{\theta}^1} \right) \geq 0$$

Therefore, the indirect effect is non-negative, making $\Delta\Lambda > 0$.

■



* This figure does not reflect risk committee and related risk management requirements or single-counterparty credit limits.
† For firms subject to Category III requirements with wSTWF of $75 billion or more, 100% LCR and NSFR requirements would apply. For firms subject to Category III requirements with less than $75 billion in wSTWF, the proposal would request comment on reducing the LCR and NSFR requirements to a level between 70-85%.
Glossary: NBA – nonbank assets; wSTWF – weighted short-term wholesale funding; AOCI – accumulated other comprehensive income; CCAR – Comprehensive Capital Analysis and Review; GSIB – global systemically important bank holding company; LCR – liquidity coverage ratio rule; NSFR – net stable funding ratio proposed rule; TLAC – total loss-absorbing capacity.

Figure 5: Federal Reserve Systemically Important Bank holding company categorization

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